Math 106 Midterm Exam #3 May 6, 2022

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $\ln(e^3)$, $e^{2\ln 3}$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [8 Points] Compute the following Derivative y' where $y = \ln\left(\dfrac{\ln x\ \sqrt{1+e^x}}{(4-x^6)^3\ e^{-\cos x}}\right)$

Do not simplify your final answer here.

2. [8 Points] Use Logarithmic Differentiation to compute $\dfrac{dy}{dx}$ where $y = x^x$

3. [12 Points] Compute each of the following Derivatives.

(a) Consider $f(x) = \sqrt{\ln x} - \ln\sqrt{x}$ Show that $f'(e^4) = \boxed{-\dfrac{1}{4e^4}}$

(b) Consider $f(x) = e^{2x} + \dfrac{1}{e^{2x}}$ Show that $f'(\ln 3) = \boxed{\dfrac{160}{9}}$. Hint: $(18)\cdot 9 = 162$

4. [8 Points]

Consider $f(x) = \sin(\ln(1+x)) - \ln(1+\sin(5x)) - e^{\cos x} - \sin\left(e^{3x}-1\right)$. Show that $f'(0) = \boxed{-7}$

5. [10 Points] Consider this function with 10 terms.

$$f(x) = e^{6x} + \frac{6}{e^{6x}} + e^{\ln 6} - \frac{6}{x} + \frac{1}{e^{6x}} - \ln\left(e^6\right) + 6e^{6x} - \frac{e}{x^6} + \frac{e^x}{e^{6x}} + \left(e^{6x}\right)\cdot\left(e^x\right)$$

Compute $f'(x)$.

6. [24 Points] Compute the following Indefinite Integrals.

(a) $\int e^{2x}\left(2+e^{2x}\right)^7\ dx$

(b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$

(c) $\int \frac{\left(1+e^{2x}\right)^2}{e^{6x}}\ dx$

7. [30 Points] Evaluate each of the following Definite Integrals. Simplify.

(a) Show that $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{e+\cos x}\ dx = \boxed{1-\ln|e-1|}$

(b) Show that $\int_0^{\ln 3} \frac{e^{2x}}{1+e^{2x}}\ dx = \boxed{\frac{1}{2}\ln 5}$

(c) Show that $\int_{e^3}^{e^8} \frac{1}{x\sqrt{1+\ln x}}\ dx = \boxed{2}$