# Exam 2 Spring 2024

1(a) $$\int_{-1}^{2} 3-4x-x^2\,dx = 3x - \frac{\cancel{4}^{2}x^2}{\cancel{2}} - \frac{x^3}{3}\Bigg|_{-1}^{2} = 3(2)-2(2)^2-\frac{2^3}{3}-\left(3(-1)-2(-1)^2-\frac{(-1)^3}{3}\right)$$

$$= 6-8-\frac{8}{3}+3+2-\frac{1}{3} = \cancel{8}-\cancel{8}+3-\frac{\cancel{9}^{3}}{\cancel{3}} = 3-3 = 0$$ Match!

1(b) $$\int_{-1}^{2} 3-4x-x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-1+\frac{3i}{n}\right)\cdot\frac{3}{n}$$

$f(x)=3-4x-x^2$

$a=-1 \quad b=2$

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$x_i = a+i\Delta x = -1+\frac{3i}{n}$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(3-4\left(-1+\frac{3i}{n}\right)-\left(-1+\frac{3i}{n}\right)^2\right)\frac{3}{n}$$

$1-\frac{6i}{n}+\frac{9i^2}{n^2}$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 3+4-\frac{12i}{n}-1+\frac{6i}{n}-\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 6-\frac{6i}{n}-\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}6-\frac{3}{n}\sum_{i=1}^{n}\frac{6i}{n}-\frac{3}{n}\sum_{i=1}^{n}\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{18}{n}\cancel{\sum_{i=1}^{n}1}^{\,n}-\frac{18}{n^2}\sum_{i=1}^{n}i-\frac{27}{n^3}\sum_{i=1}^{n}i^2$$

$$= \lim_{n\to\infty}\frac{18}{\cancel{n}}\cdot\cancel{n}-\frac{18}{\underbrace{n^2}_{n\cdot n}}\left(\frac{n(n+1)}{2}\right)-\frac{27}{\underbrace{n^3}_{n\cdot n\cdot n}}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty} 18-\frac{\cancel{18}^{9}}{\cancel{2}}\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{n+1}{n}\right)-\frac{27}{6}\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$

$$= \lim_{n\to\infty} 18-9(1)\left(\cancel{1}^{1}+\cancel{\frac{1}{n}}^{0}\right)-\frac{27}{6}(1)\left(\cancel{1}^{1}+\cancel{\frac{1}{n}}^{0}\right)\left(\cancel{2}^{2}+\cancel{\frac{1}{n}}^{0}\right)$$

$$= 18-9-\frac{27}{\cancel{6}_{3}}\cdot\cancel{2} = 9-9 = 0$$ Match!

2(a) $$\int\frac{5}{\sqrt{x}\left(3+\sqrt{x}\right)^7}dx = 5\cdot 2\int\frac{1}{u^7}du = \cancel{10}^{5}\cdot\frac{u^{-6}}{-\cancel{6}_{3}}+C = -\frac{5}{3}\left(\frac{1}{(3+\sqrt{x})^6}\right)+C$$

$u^{-7}$

$$\boxed{\begin{array}{l} u = 3+\sqrt{x} \\ du = \frac{1}{2\sqrt{x}}dx \\ 2du = \frac{1}{\sqrt{x}}dx \end{array}}$$

2(b) $\displaystyle\int \frac{\sec^2 x}{(5+\tan x)^3}\,dx = \int \frac{1}{u^3}\,du = \frac{u^{-2}}{-2} + C = -\frac{1}{2(5+\tan x)^2} + C$

$u^{-3}$

$$\boxed{\begin{array}{l} u = 5+\tan x \\ du = \sec^2 x\,dx \end{array}}$$

2(c) $\displaystyle\int \frac{x}{(x-3)^9}\,dx = \int \frac{u+3}{u^9}\,du = \int \frac{u}{u^9} + \frac{3}{u^9}\,du = \int u^{-8} + 3u^{-9}\,du$

$$\boxed{\begin{array}{l} u = x-3 \Rightarrow x = u+3 \\ du = dx \end{array}}$$

$$= \frac{u^{-7}}{-7} + 3\,\frac{u^{-8}}{-8} + C$$

$$= -\frac{1}{(x-3)^7} - \frac{3}{8(x-3)^8} + C$$

3(a) $\displaystyle\int_{\frac{\pi}{9}}^{\frac{\pi}{3}} \sin(3x) + \sqrt{3}\cos(6x)\,dx$ split $= \displaystyle\int_{\frac{\pi}{9}}^{\frac{\pi}{3}} \sin(3x)\,dx + \sqrt{3}\int_{\frac{\pi}{9}}^{\frac{\pi}{3}} \cos(6x)\,dx$

$$\boxed{\begin{array}{l} u = 3x \\ du = 3\,dx \\ \frac{1}{3}du = dx \end{array}}$$

$$\boxed{\begin{array}{l} x = \frac{\pi}{9} \Rightarrow u = 3\cdot\frac{\pi}{9} = \frac{\pi}{3} \\ x = \frac{\pi}{3} \Rightarrow u = 3\cdot\frac{\pi}{3} = \pi \end{array}}$$

$$= \frac{1}{3}\int_{\frac{\pi}{3}}^{\pi} \sin u\,du + \frac{\sqrt{3}}{6}\int_{\frac{2\pi}{3}}^{2\pi} \cos w\,dw$$

$$\boxed{\begin{array}{l} w = 6x \\ dw = 6\,dx \\ \frac{1}{6}dw = dx \end{array}}$$

$$\boxed{\begin{array}{l} x = \frac{\pi}{9} \Rightarrow u = 6\,\frac{\pi}{9} = \frac{2\pi}{3} \\ x = \frac{\pi}{3} \Rightarrow u = 6\cdot\frac{\pi}{3} = 2\pi \end{array}}$$

$$= -\frac{1}{3}\cos u\Big|_{\frac{\pi}{3}}^{\pi} + \frac{\sqrt{3}}{6}\sin w\Big|_{\frac{2\pi}{3}}^{2\pi}$$

$$= -\frac{1}{3}\underbrace{\cos\pi}_{-1} - \left(-\frac{1}{3}\underbrace{\cos\frac{\pi}{3}}_{\frac{1}{2}}\right) + \frac{\sqrt{3}}{6}\underbrace{\sin(2\pi)}_{0} - \frac{\sqrt{3}}{6}\underbrace{\sin\left(\frac{2\pi}{3}\right)}_{\frac{\sqrt{3}}{2}}$$

$+$

$$= \frac{1}{3} + \frac{1}{6} + 0 - \frac{3}{12} = \frac{4}{12} + \frac{2}{12} - \frac{3}{12} = \frac{3}{12} = \frac{1}{4}$$

Match!

3(b) $$\int_{-\pi}^{3\pi} \cos\left(\frac{x}{2}\right) dx = 2\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos u \, du = 2\sin u \Big|_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} = 2\cdot\left(\overset{-1}{\cancel{\sin\frac{3\pi}{2}}} - \overset{-1}{\cancel{\sin\left(-\frac{\pi}{2}\right)}}\right)$$

$$= 2\cdot\left(\overbrace{\cancel{-1}+\cancel{1}}^{0}\right) = 0$$ Match!

OR $= -2+2 = 0$

$$\boxed{\begin{aligned} u &= \frac{x}{2} = \frac{1}{2}\cdot x \\ du &= \frac{1}{2}dx \\ 2du &= dx \end{aligned}}$$

$$\boxed{\begin{aligned} x = -\pi &\Rightarrow u = -\frac{\pi}{2} \\ x = 3\pi &\Rightarrow u = \frac{3\pi}{2} \end{aligned}}$$

3(c) $$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\cos^3 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{(\cos x)^3} dx = -\int_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \overset{u^{-3}}{\frac{1}{u^3}} du = \cancel{-}\frac{u^{-2}}{\cancel{-}2}\Bigg|_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} = \frac{1}{2u^2}\Bigg|_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}}$$

$$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x \, dx \\ -du &= \sin x \, dx \end{aligned}}$$

$$= \frac{1}{2}\left(\frac{1}{\underbrace{\left(\frac{1}{2}\right)^2}_{\frac{1}{4}}} - \frac{1}{\underbrace{\left(\frac{\sqrt{3}}{2}\right)^2}_{\frac{3}{4}}}\right) = \frac{1}{2}\left(4 - \frac{4}{3}\right)$$

$$\boxed{\begin{aligned} x = \frac{\pi}{6} &\Rightarrow u = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \\ x = \frac{\pi}{3} &\Rightarrow u = \cos\frac{\pi}{3} = \frac{1}{2} \end{aligned}}$$

$$= \frac{1}{2}\left(\frac{12}{3} - \frac{4}{3}\right) = \frac{1}{\cancel{2}}\left(\frac{\overset{4}{\cancel{8}}}{3}\right) = \frac{4}{3}$$ Match!

4. $$\int_{-1}^{3} |x-2|+1 \, dx = \int_{-1}^{2} \overset{-x+2}{\cancel{|x-2|}}+1 \, dx + \int_{2}^{3} \overset{x-2}{\cancel{|x-2|}}+1 \, dx$$

$$= \int_{-1}^{2} -x+3 \, dx + \int_{2}^{3} x-1 \, dx$$

Cases

$$|x-2| = \begin{cases} x-2 & \text{if } x-2 \geq 0 \\ -(x-2) & \text{if } x-2 < 0 \end{cases}$$

$$= \begin{cases} x-2 & \text{if } x \geq 2 \\ -x+2 & \text{if } x < 2 \end{cases}$$

$$= -\frac{x^2}{2} + 3x \Big|_{-1}^{2} + \frac{x^2}{2} - x \Big|_{2}^{3}$$

$$= -\frac{4}{2} + 6 - \left(-\frac{\overset{1}{(-1)^2}}{2} - 3\right) + \frac{9}{2} - 3 - \overset{2-2}{\cancel{\left(\frac{4}{2} - 2\right)}}$$

$$= -2 + 6 + \frac{1}{2} + \cancel{3} + \frac{9}{2} - \cancel{3}$$

$$= 4 + \frac{10}{2} = 4 + 5 = 9$$ Match!

5. $f'(x) = \frac{1}{x^3\sqrt{3+\frac{6}{x^2}}}$ and $f(1) = -\frac{5}{2}$

$$f(x) = \int f'(x)\,dx = \int \frac{1}{x^3\sqrt{3+\frac{6}{x^2}}}\,dx = -\frac{1}{12}\int \frac{1}{\sqrt{u}}\,du = -\frac{1}{12}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$u^{-\frac{1}{2}}$

$$\boxed{\begin{array}{l} u = 3+\frac{6}{x^2} \\ du = -\frac{12}{x^3}\,dx \\ -\frac{1}{12}\,du = \frac{1}{x^3}\,dx \end{array}}$$

$$= -\frac{1}{\cancel{12}_{6}}\cdot\cancel{2}\sqrt{u} + C = -\frac{1}{6}\sqrt{3+\frac{6}{x^2}} + C$$

Use Initial Value

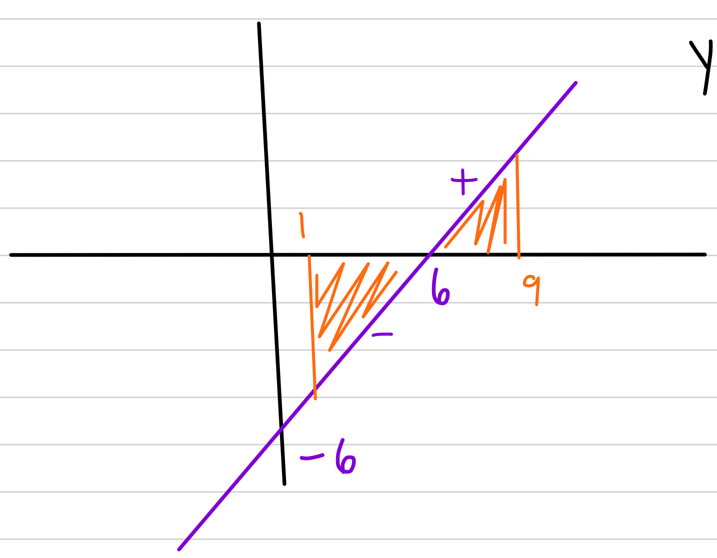
$$f(1) = -\frac{1}{6}\cancel{\sqrt{3+6}}^{\sqrt{9}=3} + C \overset{\text{set}}{=} -\frac{5}{2}$$

$$-\frac{1}{2} + C = -\frac{5}{2} \Rightarrow C = -\frac{4}{2} = -2$$

$+\frac{1}{2}$ $\quad$ $+\frac{1}{2}$

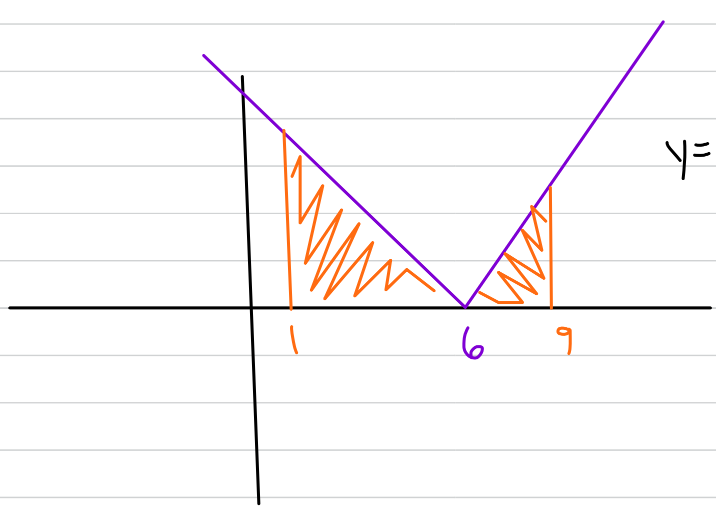
Finally, $f(x) = -\frac{1}{6}\sqrt{3+\frac{6}{x^2}} - 2$

6(a) $\int_1^9 x-6\,dx$ = Net Area




$y = x-6$

6(b) $\int_1^9 |x-6|\,dx$




$y = |x-6|$

6(c) $\int_{-1}^7 x^2-7x+10\,dx$ = Net Area

Factor
$x^2-7x+10 = (x-2)(x-5) = 0$

$x=2 \quad x=5$

Parabola "zeroes"