Math 106 Midterm Exam #2 April 8, 2022

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [20 Points] Compute and show that $\int_{-1}^{2} 2 - 2x - x^2 \, dx = \boxed{0}$

using two different methods:

(a) Fundamental Theorem of Calculus

(b) Limit Definition of the Definite Integral.

2. [32 Points] Evaluate each of the following Indefinite Integrals. Simplify.

(a) $\int \frac{x^6}{(8 - x^7)^5} \, dx$

(b) $\int 4 \sin x \cdot \cos^3 x \, dx$

(c) $\int \sec^2(1 - 7x) \, dx$

(d) $\int x \cdot (x + 7)^6 \, dx$

3. [32 Points] Evaluate each of the following Definite Integrals. Simplify.

(a) Show that $\displaystyle\int_1^4 \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}}\,dx = \boxed{\frac{8}{3}}$

(b) Show that $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan^3 x}\,dx = \boxed{\frac{1}{3}}$

(c) Show that $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos(4x)\,dx = \boxed{-\frac{\sqrt{3}}{4}}$

(d) Show that $\displaystyle\int_{2\pi}^{6\pi} \sin\left(\frac{x}{6}\right)\,dx = \boxed{9}$

4. [8 Points] Compute the following Definite Integral.

Show that $\displaystyle\int_{-1}^{3} |x-2|+1 = \boxed{9}$

5. [8 Points] Compute $f(x)$ where $f'(x) = \dfrac{1}{\sqrt{x}\,\sqrt{2+\sqrt{x}}}$ and $f(4) = -5$