Math 106 Exam 1
February 23, 2024

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [21 Points] Differentiate each of the following functions. Do **NOT** simplify your answers.

(a) $f(x) = \sin\left(\frac{\pi}{4}\right) + \sqrt{\sin\sqrt{x}}$ (b) $f(x) = \cos(\sin x)$

(c) $f(x) = \cos x \cdot \sin x$ (d) $f(x) = \tan^8\left(\frac{4}{x^5}\right) \overset{\text{prep}}{=} \left(\tan\left(\frac{4}{x^5}\right)\right)^8$

(e) $f(x) = \dfrac{1}{\cos x}$ (f) $f(x) = \cos\left(\frac{1}{x}\right)$

2. [20 Points] Derivatives

(a) Let $f(x) = \cos(7x) + \cos(6x) + \sin(3x) + \sin(4x)$ Show that $f'\left(\frac{\pi}{6}\right) = \boxed{\frac{3}{2}}$

(b) Let $H(x) = \cos^2(2x) + \sin(6x) + 2\sin x$ Show that $H'\left(\frac{\pi}{6}\right) = \boxed{-6}$

3. [24 Points] Compute the following **Most General Antiderivatives.**

(a) $\displaystyle\int \frac{5}{6}x^5 + x^{\frac{5}{6}} + \frac{1}{x^{\frac{5}{6}}} + \frac{6}{5} + \frac{5}{6x^6} - \frac{6}{x^5}\,dx$

(b) $\displaystyle\int \sec^2 x - 8\cos x + \sin x + \frac{\sec x \tan x}{7}\,dx$

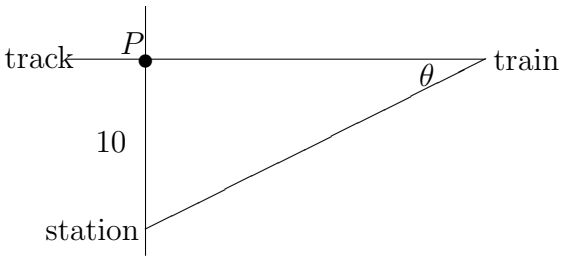
(c) $\displaystyle\int \left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)\,dx$

(d) $\displaystyle\int \frac{x^4 - 5x^2 + \sqrt{x} + 7}{x^2}\,dx$

4. [15 Points] Consider a point P on a train track. Suppose a train depot station is 10 feet directly south from this point P. The train is travelling east at 6 feet per second. Consider the angle as shown in the diagram. How fast is this angle changing when the distance between the train and the station is 20 feet?

• Diagram

The picture at arbitrary time t is:



5. [8 Points] Consider a function f such that $f'(x) = \dfrac{32}{x^3} - \dfrac{1}{\sqrt{x}} + 2$ and $f(4) = -6$

Compute $f(x)$.

6. [12 Points] For the following, use $a(t) = -32$ feet per second squared as acceleration due to gravity on the falling body.

A ball is thrown *upwards* from the edge of the top of a building that is 80 feet tall with an initial velocity of 64 feet per second. Answer the following questions.

• **When** does the ball reach its Maximum Height?

• **What** is the Maximum Height reached by the ball?

• **When** does the ball strike the ground?

• **What** is the Velocity of the ball at impact with the ground?

HINT: $-16t^2 + 64t + 80 = -16(t^2 - 4t - 5)$