Exam 1 Spring 2022

1(a) $f(x) = \overset{\text{constant}}{\cos\pi} + \sqrt{\cos\sqrt{x}} = \cos\pi + \left(\cos\sqrt{x}\right)^{1/2}$

$f'(x) = 0 + \boxed{\frac{1}{2}\left(\cos\sqrt{x}\right)^{-1/2}\cdot\left(-\sin\sqrt{x}\right)\cdot\frac{1}{2\sqrt{x}}}$ Do Not Need to Simplify

1(b) $f(x) = \cos(\sin x)$

$f'(x) = \boxed{-\sin(\sin x)\cdot\cos x}$

1(c) $f(x) = \cos x\cdot\sin x$

$f'(x) = \boxed{\cos x\cdot\cos x + \sin x(-\sin x)} \overset{\text{OR}}{=} \cos^2 x - \sin^2 x$

1(d) $f(x) = \cos^5\left(\frac{7}{x^6}\right) \overset{\text{prep}}{=} \left(\cos\left(\frac{7}{x^6}\right)\right)^5 = \left(\cos\left(7x^{-6}\right)\right)^5$

$f'(x) = \boxed{5\left(\cos\left(\frac{7}{x^6}\right)\right)^4\cdot\left(\cancel{-}\sin\left(\frac{7}{x^6}\right)\right)\cdot\left(\cancel{-}42x^{-7}\right)}$

OR $= \dfrac{210\cos^4\left(\frac{7}{x^6}\right)\cdot\sin\left(\frac{7}{x^6}\right)}{x^7}$

1(e) $f(x) = \left(\dfrac{\cos(7x)}{\tan(3x)}\right)^{7/8}$

$f'(x) = \boxed{\frac{7}{8}\left(\frac{\cos(7x)}{\tan(3x)}\right)^{-1/8}\cdot\frac{\tan(3x)\cdot(-\sin(7x))\cdot 7 - \cos(7x)\cdot\sec^2(3x)\cdot 3}{(\tan(3x))^2}}$

OR $\tan^2(3x)$

2(a) $f(x) = \tan(2x)\cdot\sin(2x)$

$f'(x) = \tan(2x)\cdot\cos(2x)\cdot 2 + \sin(2x)\cdot\sec^2(2x)\cdot 2$

$f'\left(\frac{\pi}{6}\right) = \tan\left(2\cdot\frac{\pi}{6}\right)\cos\left(2\cdot\frac{\pi}{6}\right)\cdot 2 + \sin\left(2\cdot\frac{\pi}{6}\right)\cdot\sec^2\left(2\cdot\frac{\pi}{6}\right)\cdot 2$ ($\pi/3$)

$= \underbrace{\tan\left(\frac{\pi}{3}\right)}_{\sqrt{3}}\cdot\underbrace{\cos\left(\frac{\pi}{3}\right)}_{1/2}\cdot 2 + \underbrace{\sin\left(\frac{\pi}{3}\right)}_{\frac{\sqrt{3}}{2}}\cdot\left(\underbrace{\sec\left(\frac{\pi}{3}\right)}_{(2)^2}\right)^2\cdot 2$

$\sec\frac{\pi}{3} = \frac{1}{\cos\pi/3} = \frac{1}{(1/2)} = 2$

$= \sqrt{3}\cdot\frac{1}{\cancel{2}}\cdot\cancel{2} + \frac{\sqrt{3}}{\cancel{2}}\cdot 4\cdot\cancel{2} = \sqrt{3} + 4\sqrt{3} = \boxed{5\sqrt{3}}$

2(b) $f(x) = 2\sin^2 x = 2(\sin x)^2$

$f'(x) = 2 \cdot 2(\sin x)^1 \cdot \cos x = 4 \sin x \cdot \cos x$

$f'\left(\frac{\pi}{4}\right) = 4 \cancel{\sin\left(\frac{\pi}{4}\right)}^{\frac{\sqrt{2}}{2}} \cdot \cancel{\cos\left(\frac{\pi}{4}\right)}^{\frac{\sqrt{2}}{2}} = \cancel{4} \cdot \frac{\sqrt{2}}{\cancel{2}} \cdot \frac{\sqrt{2}}{\cancel{2}} = \sqrt{2}\sqrt{2} = 2^{1/2} \cdot 2^{1/2} = 2^1 = \boxed{2}$

2(c) $H(x) = \cos(4x)$

$H'(x) = -\sin(4x) \cdot 4$

$H'\left(\frac{\pi}{3}\right) = -\cancel{\sin\left(\frac{4\pi}{3}\right)}^{-\frac{\sqrt{3}}{2}} \cdot 4 = \cancel{-}\left(\cancel{-}\frac{\sqrt{3}}{\cancel{2}}\right) \cdot \cancel{4}^{2} = \boxed{2\sqrt{3}}$

$H'\left(\frac{\pi}{8}\right) = -\sin\left(\frac{\cancel{4}\pi}{\cancel{8}_{2}}\right) \cdot 4 = -\cancel{\sin\left(\frac{\pi}{2}\right)}^{1} \cdot 4 = \boxed{-4}$

3(a) $\int \frac{5}{6}x^5 + x^{\frac{5}{6}} + \frac{1}{x^{\frac{5}{6}}} + \frac{6}{5} + \frac{5}{6x^6} - \frac{6}{x^5}\,dx$

prep

$= \int \frac{5}{6}x^5 + x^{\frac{5}{6}} + x^{\frac{-5}{6}} + \frac{6}{5} + \frac{5}{6}x^{-6} - 6x^{-5}\,dx$

$= \frac{5}{6}\frac{x^6}{6} + \frac{x^{\frac{11}{6}}}{\frac{11}{6}} + \frac{x^{1/6}}{\frac{1}{6}} + \frac{6}{5}x + \frac{\cancel{5}}{6}\left(\frac{x^{-5}}{-\cancel{5}}\right) \cancel{-} 6\frac{x^{-4}}{\cancel{-}4} + C$

$= \boxed{\frac{5}{36}x^6 + \frac{6}{11}x^{11/6} + 6x^{1/6} + \frac{6}{5}x - \frac{1}{6x^5} + \frac{3}{2x^4} + C}$

3(b) $\int \sec^2 x - 8\cos x + \sin x + \frac{\sec x \cdot \tan x}{7}\,dx$

$= \boxed{\tan x - 8\sin x - \cos x + \frac{1}{7}\cdot \sec x + C}$

3(c) $\int\left(x^2 + \frac{1}{x^2}\right)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$ FOIL $= \int x^2\sqrt{x} + \frac{x^2}{\sqrt{x}} + \frac{\sqrt{x}}{x^2} + \frac{1}{x^2\sqrt{x}}\,dx$

$= \int x^{5/2} + x^{3/2} + x^{-3/2} + x^{-5/2}\,dx$

$= \frac{x^{7/2}}{7/2} + \frac{x^{5/2}}{5/2} + \frac{x^{-1/2}}{-1/2} + \frac{x^{-3/2}}{-3/2} + C$

$= \boxed{\frac{2}{7}x^{7/2} + \frac{2}{5}x^{5/2} - \frac{2}{x^{1/2}} - \frac{2}{3x^{3/2}} + C}$

3(d) $\displaystyle\int \frac{x^2+\sqrt{x}}{x^{3/7}}\,dx = \int \frac{x^2}{x^{3/7}} + \frac{\sqrt{x}}{x^{3/7}}\,dx = \int \frac{x^{14/7}}{x^{3/7}} + \frac{x^{7/14}}{x^{6/14}}\,dx$

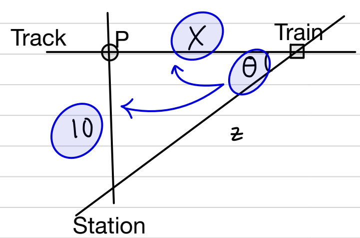
split

$x^{1/2}$

$\displaystyle = \int x^{11/7} + x^{1/14}\,dx = \frac{x^{18/7}}{18/7} + \frac{x^{15/14}}{15/14} + C$

$$= \boxed{\frac{7}{18}x^{18/7} + \frac{14}{15}x^{15/14} + C}$$

4. Diagram → given



Variables

Let x = Distance between train and Point P

z = Distance between train and station

θ = Angle between track and line connecting the Train the Station

Given $\frac{dx}{dt} = 6$ ft/sec

Find $\frac{d\theta}{dt} = ?$ when $t = 2$ sec. $\Rightarrow x = 6\,\text{ft}/\cancel{\text{sec}} \cdot (2\,\cancel{\text{sec}}) = 12$ feet

Equation

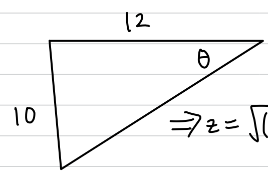
$\tan\theta = \frac{10}{x}$

Differentiate

$\frac{d}{dt}\left(\tan\theta\right) = \frac{d}{dt}\left(10x^{-1}\right)$

$(\sec\theta)^2$ → $\sec^2\theta \cdot \frac{d\theta}{dt} = -10x^{-2} \cdot \frac{dx}{dt}$

Extra Solvable Information



$\Rightarrow z = \sqrt{(10)^2+(12)^2} = \sqrt{100+144} = \sqrt{244}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{H}{A} = \frac{\sqrt{244}}{12}$

$\frac{A}{H}$

Substitute

?

$\left(\frac{\sqrt{244}}{12}\right)^2 \cdot \frac{d\theta}{dt} = -\frac{10}{(12)^2} \cdot 6$

Solve $\quad \frac{244}{\cancel{(12)^2}} \cdot \frac{d\theta}{dt} = -\frac{60}{\cancel{(12)^2}} \qquad \frac{\cancel{(12)^2}}{244}$

$$\frac{d\theta}{dt} = \frac{-60}{244} = \frac{-15}{61} \quad \text{Radians per Second}$$

negative, decreasing makes sense

Answer $\boxed{\text{This angle is decreasing at a rate of } 15/61 \text{ Radians every second at this Moment.}}$

5. $f'(x) = 2\sin x - 5\cos x \qquad f(\pi) = -6$

Antidifferentiate

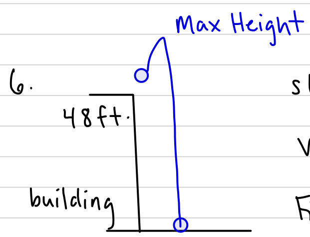
$$f(x) = \int f'(x)\,dx = \int 2\sin x - 5\cos x\,dx = -2\cos x - 5\sin x + C$$

Use given value to solve for $+C$

$$f(\pi) = -2\cancel{\cos\pi}^{\,-1} - 5\cancel{\sin\pi}^{\,0} + C \overset{\text{set}}{=} -6 \quad \hookrightarrow \quad 2 - 0 + C = -6 \Rightarrow C = -8$$

Finally, $f(x) = \boxed{-2\cos x - 5\sin x - 8}$

6.



$s(0) = 48$ ft.

$v(0) = 32$ ft/sec

Find $t_{max} = ?$

Find Max Height $\hookrightarrow s(t_{max})$

$a(t) = -32$

$v(t) = -32t + \cancel{v_0}^{\,32}$

$= -32t + 32$

$s(t) = -16t^2 + \cancel{v_0}^{\,32}t + \cancel{s_0}^{\,48}$

$= -16t^2 + 32t + 48$

Max Height is when $v(t) = 0$

$v(t) = -32t + 32 \overset{\text{set}}{=} 0 \quad \hookrightarrow \quad$ Solve $32t = 32 \Rightarrow \boxed{t_{max} = 1 \text{ second}}$

$\boxed{\text{Max Height occurs at } t = 1 \text{ second.}}$

Max Height occurs when $t_{max} = 1$

$\Rightarrow s(t_{max}) = s(1) = -16 + 32 + 48 = 16 + 48 = \boxed{64 \text{ feet}}$

$\boxed{\text{Max Height is 64 feet}}$

Velocity at time $t = 2 \Rightarrow v(2) = -32(2) + 32 = -64 + 32 = \boxed{-32 \text{ ft/sec}}$

Makes Sense $\hookrightarrow$ Ball is travelling Down so Velocity negative.