**Definite Integral Limit Definition Formulas**

Definition: the Definite Integral of a function $f$ from $x = a$ to $x = b$ is given by

$$(\bullet) \int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \ldots + f(x_i)\Delta x + \ldots + f(x_n)\Delta x]$$

Note: The definite integral is a limit of a sum! Just think about this formula as

*the* **limiting value** *of the sum of the areas of finitely many (n) approximating rectangles.*

To compute definite integrals the *long (limit) way*, follow these steps:

Step 1: Given the integral $\int_a^b f(x)\, dx$, **pick off** or **identify** the **integrand** $f(x)$, and **limits of integration** $a$ and $b$.

Step 2: Compute $\boxed{\Delta x = \frac{b-a}{n}}$. This width of each partitioned interval should be in terms of $n$.

Step 3: Compute $\boxed{x_i = a + i\Delta x}$. Leave the $i$ as your counter. You have the left-most endpoint $a$ from Step 1. You have width $\Delta x$ from Step 2. This endpoint $x_i$ should be in terms of $i$ and $n$.

Step 4: Plug $x_i$ and $\Delta x$ into the formula $(\bullet)$ above. Here it is again:

$$(\bullet) \quad \boxed{\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x} \longleftarrow \textbf{MEMORIZE!}$$

Step 5: Use the following formulas for sum of integers $i$ and finish evaluating the limit in $n$.

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Note: your final answer for the definite integral should be a **number** after you finish the limit.