Math 106, Spring 2023

Worksheet 5, Tuesday, March 7, 2023

NOTE: Unless instructions specify to use the Limit Definition of the Definite Integral, you may use the *Quicker* Fundamental Theorem of Calculus, Part II.

Compute each of the following Definite Integrals. Simplify.

1. $\displaystyle\int_0^{\frac{\pi}{3}} \sec^2\theta \, d\theta$

2. $\displaystyle\int_{-\pi}^{\frac{\pi}{3}} 7\cos x \, dx$

3. $\displaystyle\int_{-2}^{-1} x - \frac{5}{x^3} \, dx$

4. $\displaystyle\int_0^{\frac{\pi}{6}} (\tan x + \sec x)\sec x \, dx$

5. $\displaystyle\int_1^2 \left(x - \frac{1}{x}\right)^2 \, dx$

6. $\displaystyle\int_0^{16} \frac{1}{x^{\frac{3}{4}}} - \frac{2}{\sqrt{x}} \, dx$

7. $\displaystyle\int_1^4 \frac{\sqrt{x} - x^2}{x} \, dx$

8. Show that $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{3 + \cos^2 x}{\cos^2 x} \, dx = \boxed{3(\sqrt{3} - 1) + \frac{\pi}{12}}$

9. Show that $\displaystyle\int_{-\pi}^{\pi} \sin x \, dx = \boxed{0}$. Explain why that makes sense?

10(a) Write the function cases definition for $f(x) = |x|$.

10(b) Compute $\displaystyle\int_{-2}^{1} |x| \, dx$. Recall how the absolute value is defined above in (a). Then draw the bounded region and use *Area Interpretation* to confirm your answer.

11. Compute $\displaystyle\int_2^5 x^2 \, dx$ using each of the following two methods:
(a) The Fundamental Theorem of Calculus.
(b) The *Limit Definition* of the Definite Integral

12(a) Write the function cases definition for $f(x) = |x - 5|$.

12(b) Compute $\displaystyle\int_4^7 |x - 5| \, dx$. Again, draw the bounded region and use *Area Interpretation* to confirm your answer.

Turn in your own solutions.