Math 106, Spring 2023

Worksheet 4, Tuesday, February 28th, 2023

Manual Area Estimates

1. Consider $f(x) = x^2 + 1$. Estimate the Area bounded above by the graph of $f(x) = x^2 + 1$ and above the x-axis between $x = 0$ to $x = 2$ using TWO Approximating Rectangles and *Right* endpoints. Sketch the graph and the rectangles.

2. Repeat the same process in 1. above using FOUR rectangles and *Right* endpoints. Sketch the graph and the rectangles.

(*) Compare your answers in 1-2 for fun. Which ones are an overestimate or underestimate of the actual bounded area?

Manual Area Computations

3. Compute by hand, manually, the Area bounded above by the graph of $y = x$ and below by $y = 0$ and between $x = 0$ and $x = 6$. Sketch the graph and shade the bounded region.

4. Compute by hand, manually, the Area bounded above by the graph of $y = x + 3$ and below by $y = 0$ and between $x = 0$ and $x = 4$. Sketch the graph and shade the bounded region.

Function Evaluation For problems 5-8, i and n are some constants.

5. Consider $f(x) = x$. Compute and simplify $f\left(\frac{5i}{n}\right)$ and $f\left(2 + \frac{5i}{n}\right)$.

6. Consider $f(x) = 3x - 4$. Compute and simplify $f\left(\frac{8i}{n}\right)$ and $f\left(3 + \frac{8i}{n}\right)$.

7. Consider $f(x) = x^2 + 5$. Compute and simplify $f\left(\frac{2i}{n}\right)$ and $f\left(4 + \frac{2i}{n}\right)$.

8. Consider $f(x) = x^2 - 2x + 7$. Compute and simplify $f\left(\frac{6i}{n}\right)$ and $f\left(-1 + \frac{6i}{n}\right)$.

Limit Finishes Compute and Simplify the following Limits. Justify, using arrows.

9. $\lim_{n\to\infty} 3 =$

10. $\lim_{n\to\infty} \frac{1}{n} =$

11. $\lim_{n\to\infty} 1 + \frac{1}{n} =$

12. $\lim_{n\to\infty} \frac{n+1}{n} =$

13. $\lim_{n\to\infty} \frac{n+3}{n} =$

14. $\lim_{n\to\infty} \frac{2n+1}{n} =$

15. $\lim_{n\to\infty} \frac{n(n+1)}{n^2} =$

16. $\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{n^3} =$

17. $\lim_{n\to\infty} 3 - \left(\frac{4}{n^2}\right) \cdot \left(\frac{n(n+1)}{2}\right) - \left(\frac{12}{n^3}\right) \cdot \frac{n(n+1)(2n+1)}{6} =$

Summation Algebra Rules $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$

Specific Constant Rule for summing 1 n times

$$\sum_{i=1}^{n} 1 = \underbrace{1 + 1 + 1 + \ldots + 1}_{\text{n copies}} = n$$

Sum/Difference Rule

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

Constant Multiple Rule

$$\sum_{i=1}^{n} \text{constant} \cdot a_i = \text{constant} \sum_{i=1}^{n} a_i$$

Constant Rule

$$\sum_{i=1}^{n} \text{constant} = \text{constant} \sum_{i=1}^{n} 1 = \text{constant} \cdot n$$

18. Simplify $\sum_{i=1}^{n} 6$

19. Simplify $\sum_{i=1}^{n} (-3)$

20. Simplify. Show that $\sum_{i=1}^{n} \left(\frac{6i}{n} - 5\right) \cdot \left(\frac{6}{n}\right) = \left(\frac{36}{n^2} \sum_{i=1}^{n} i\right) - 30$

21. Simplify. Show that $\sum_{i=1}^{n} \left(1 + \frac{3i}{n}\right)^2 \cdot \left(\frac{3}{n}\right) = 3 + \frac{18}{n^2} \sum_{i=1}^{n} i + \frac{27}{n^3} \sum_{i=1}^{n} i^2$

Turn in your own solutions.