

Worksheet 2, Tuesday, February 14th, 2023
Antiderivatives

Definition: Take F, f functions defined on an interval I and suppose that $F'(x) = f(x)$ on I . Then

- $F(x)$ is called **AN** Antiderivative of $f(x)$
- $F(x) + C$ is called **the Most General** Antiderivative of $f(x)$, where C is any constant Real Number.

We will use the notation $\int f(x) dx$ to denote the Most General Antiderivative. The symbols \int and dx are partnered-together instructions to compute the Most General Antiderivative.

For example: $\int x^7 dx = \frac{x^8}{8} + C$ where $+C$ represent all possible constants.

This means that $\frac{d}{dx} \left(\frac{x^8}{8} + C \right) = x^7$ which looks correct to us.

Note that $\frac{x^8}{8} + 3$ is **an** antiderivative of x^7 . So is $\frac{x^8}{8} + 2022$ as well as $\frac{x^8}{8} - 5$ and $\frac{x^8}{8} + \sqrt{3}$.

Hint: if you ever want to know whether you found the correct antiderivative, take the derivative of your answer and check that you return to the original function.

1. Write a *general power rule* for $\int x^n dx$ where n is any real number with $n \neq -1$.
 (We will learn the $n = -1$ case at the very end of this semester.)

Compute each of the following Most General Antiderivatives

2. $\int \sqrt{x} dx$ 3. $\int \frac{1}{x^9} dx$ 4. $\int \frac{1}{\sqrt{x}} dx$ 5. $\int \frac{1}{x^{\frac{3}{7}}} dx$
 6. $\int \cos x dx$ 7. $\int \sin x dx$ 8. $\int \sec^2 x dx$ 9. $\int \sec x \tan x dx$

For #10 – 11, consider $f(x) = \frac{3}{4}x + x^{\frac{3}{4}} - \frac{1}{x^{\frac{3}{4}}} + \frac{4}{3} + \frac{1}{x^{\frac{4}{3}}} + \frac{3}{4x^4} - \frac{4}{x^3}$ Hint: prep $\frac{1}{x^a} = x^{-a}$

10. Compute $f'(x) = \frac{d}{dx} \left(\frac{3}{4}x + x^{\frac{3}{4}} - \frac{1}{x^{\frac{3}{4}}} + \frac{4}{3} + \frac{1}{x^{\frac{4}{3}}} + \frac{3}{4x^4} - \frac{4}{x^3} \right)$

11. Compute $\int f(x) dx = \int \frac{3}{4}x + x^{\frac{3}{4}} - \frac{1}{x^{\frac{3}{4}}} + \frac{4}{3} + \frac{1}{x^{\frac{4}{3}}} + \frac{3}{4x^4} - \frac{4}{x^3} dx$

Find the Most General Antiderivative of the following functions. You may need to use some Algebra to *prep* the function pieces for each *Power Rule* and/or *Sum or Difference Rule(s)*.

12. $\int x^3(1 + x^2) dx$

13. $\int \frac{x + \sqrt{x} + 7}{x^3} dx$

14. $\int x^2 + x(1 + x)^2 dx$

15. $\int -3 \cos x - \sec^2 x - 7 \sec x \tan x - \sin x dx$

16. Consider the curve $y = 2x + \sin x$. Explain why the tangent lines of this curve are never horizontal.

Find the indicated functions $f(x)$ that also satisfies the given conditions:

17. $f'(x) = 2 + \sin x$ and $f\left(\frac{\pi}{2}\right) = 3$.

18. $f'(x) = x^2 + 1$ and $f(1) = 3$.

19. $f'(x) = x(2 + \sqrt{x})$ and $f(4) = 30$.

20. $f''(x) = \frac{1}{\sqrt{x}} + 3x^2$ and $f'(1) = 2$, $f(1) = 0$.

21. Can you use a *guess and check* approach to compute the function $f(x)$ where $f'(x) = \sin(3x)$? Check your answer. How? Think about *reversing the Chain Rule here...*

22. CHALLENGE: Can you use a *guess and check* approach to compute the function $f(x)$ where $f'(x) = \frac{\sec x \tan x}{\sqrt{\sec x + 8}}$ and $f(0) = 7$? Check your answer.

Turn in your own solutions.