Worksheet 3 Answer Key

1(a) $f(x) = \sin\left(\cos^6\left(\frac{9}{x^8}\right)\right) \overset{\text{prep}}{=} \sin\left(\left[\cos\left(\frac{9}{x^8}\right)\right]^6\right)$

$9x^{-8}$

1 2 3 4

$$f'(x) = \boxed{\cos\left(\cos^6\left(\frac{9}{x^8}\right)\right) \cdot 6\left(\cos\left(\frac{9}{x^8}\right)\right)^5 \cdot \left[-\sin\left(\frac{9}{x^8}\right)\right] \cdot \left(-72x^{-9}\right)}$$

1(b) $y = \tan\left(\frac{9}{\sin x}\right) \overset{\text{prep}}{=} \tan\left[9(\sin x)^{-1}\right]$

1 2 3

$$y' = \boxed{\sec^2\left(\frac{9}{\sin x}\right) \cdot 9\left[-(\sin x)^{-2}\right] \cdot \cos x}$$

OR Chain Rule + Quotient Rule

$$y' = \sec^2\left(\frac{9}{\sin x}\right) \cdot \left(\frac{\sin x \cdot 0 - 9 \cdot \cos x}{\sin^2 x}\right) = \boxed{\sec^2\left(\frac{9}{\sin x}\right) \cdot \left(\frac{-9\cos x}{\sin^2 x}\right)}$$

Match. O.K.

1(c) $g(t) = \dfrac{8 + \sec(7t^2)}{9 + \cos t}$

$$g'(t) = \boxed{\frac{(9+\cos t)\left(\sec(7t^2)\cdot\tan(7t^2)\cdot(14t)\right) - \left(8+\sec(7t^2)\right)\cdot(-\sin t)}{(9+\cos t)^2}}$$

Do Not Simplify

2. $f(x) = \cos^2(2x) + \tan(2x) + \sin(6x) + \sqrt{3}\cdot x$ Find $f'\left(\frac{\pi}{6}\right)$

prep $(\cos(2x))^2$

$$f'(x) = 2\cos(2x)(-\sin(2x))\cdot 2 + \sec^2(2x)\cdot 2 + 6\cos(6x) + \sqrt{3}$$

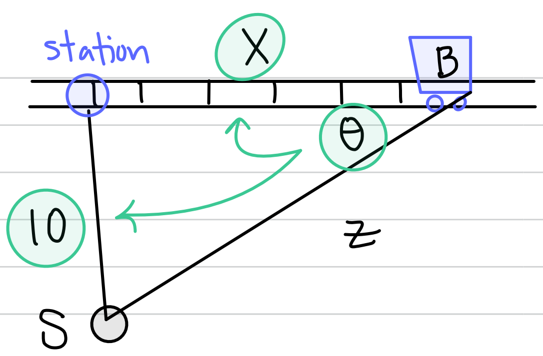
$$= -4\cos(2x)\sin(2x) + 2\sec^2(2x) + 6\cos(6x) + \sqrt{3}$$

$$f'\left(\frac{\pi}{6}\right) = -4\cos\left(\cancel{2}\cdot\frac{\pi}{\cancel{6}_3}\right)\sin\left(\cancel{2}\frac{\pi}{\cancel{6}_3}\right) + 2\left(\sec\left(\cancel{2}\cdot\frac{\pi}{\cancel{6}_3}\right)\right)^2 + 6\cos\left(\cancel{6}\cdot\frac{\pi}{\cancel{6}}\right) + \sqrt{3}$$

$$= -4\,\underbrace{\cancel{\cos\left(\frac{\pi}{3}\right)}}_{\frac{1}{2}} \cdot \underbrace{\cancel{\sin\left(\frac{\pi}{3}\right)}}_{\frac{\sqrt{3}}{2}} + 2\left(\underbrace{\cancel{\sec\left(\frac{\pi}{3}\right)}}_{(2)^2}\right)^2 + 6\,\underbrace{\cancel{\cos(\pi)}}_{-1} + \sqrt{3}$$

$$= -\cancel{4}\left(\frac{1}{\cancel{2}}\right)\left(\frac{\sqrt{3}}{\cancel{2}}\right) + 2\cdot 4 - 6 + \sqrt{3} = -\cancel{\sqrt{3}} + 8 - 6 + \cancel{\sqrt{3}} = 2$$

3. Diagram



Variables Let $x =$ Distance between Train and Station

$z =$ Distance between Bob (on Train) and Sally

$\theta =$ Angle of Bob's Head Rotation from Track Line

Given $\frac{d\theta}{dt} = -2$ Radians / second

Find $\frac{dx}{dt} = ?$ when $z = 13$ meters

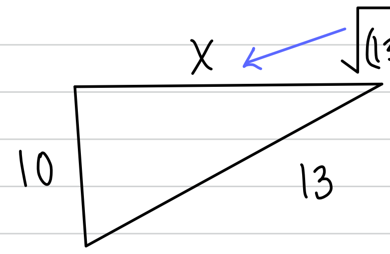
Equation $\tan\theta = \frac{10}{x}$

Differentiate $\frac{d}{dt}\left(\tan\theta\right) = \frac{d}{dt}\left(10x^{-1}\right)$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = -10x^{-2}\frac{dx}{dt}$$

$$(\sec\theta)^2 \frac{d\theta}{dt} = \frac{-10}{x^2} \cdot \frac{dx}{dt}$$

Extra Solvable Information



$x = \sqrt{(13)^2 - (10)^2} = \sqrt{169 - 100} = \sqrt{69}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{A}{H}\right)} = \frac{H}{A} = \frac{13}{\sqrt{69}}$$

Substitute

$$\left(\frac{13}{\sqrt{69}}\right)^2 \cdot (-2) = -\frac{10}{(\sqrt{69})^2} \cdot \frac{dx}{dt}$$

$$\frac{169}{69}(-2) = \frac{-10}{69} \cdot \frac{dx}{dt}$$

Solve

$$\frac{dx}{dt} = \frac{169}{\cancel{69}}\cancel{(-2)} \cdot \frac{\cancel{69}}{\cancel{-10}_{5}} = \frac{169}{5} \text{ m/sec}$$

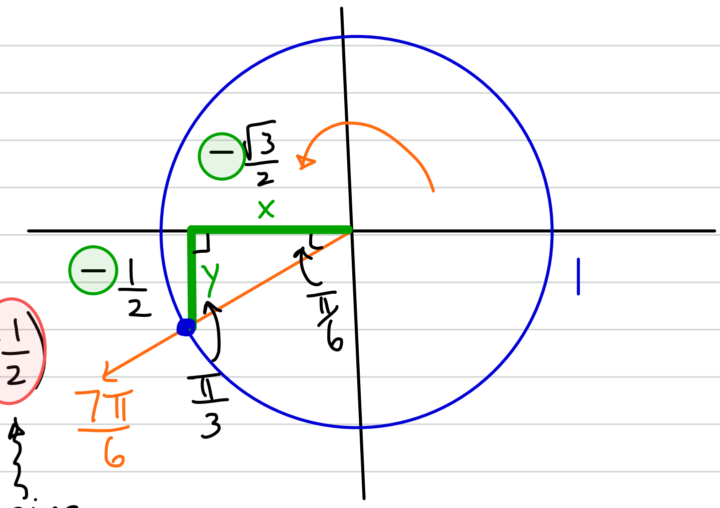
Answer

$\boxed{\text{The train is travelling } \frac{169}{5} \text{ meters every second at that moment.}}$

4. $\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

Point $(x,y) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

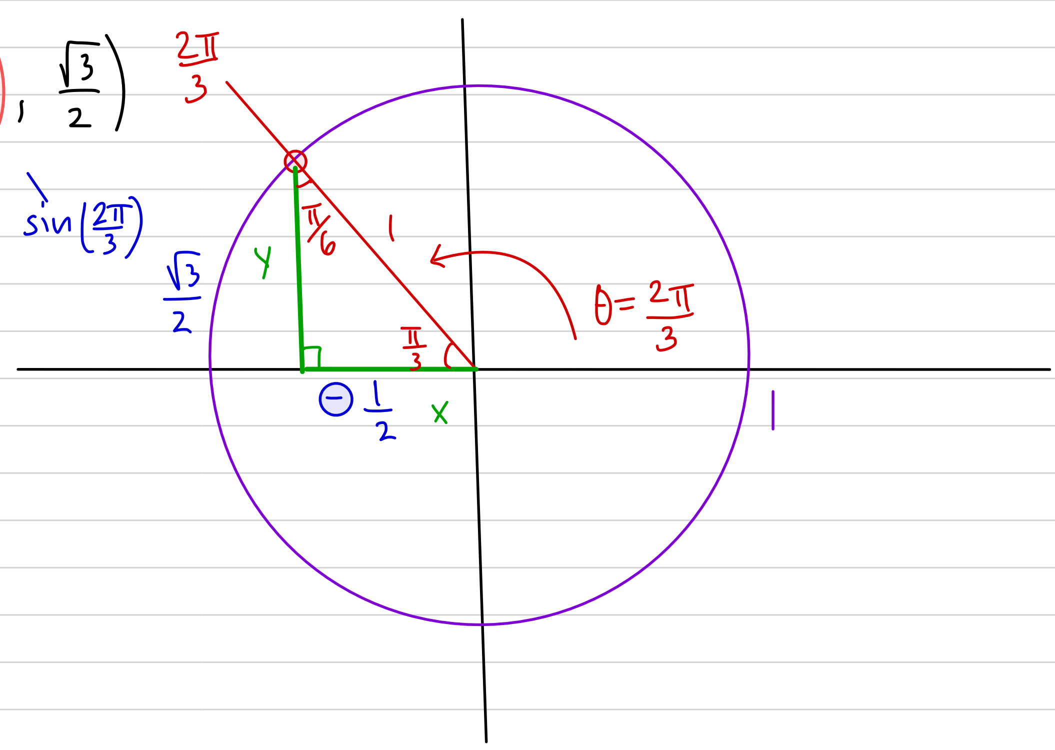
cosine sine



5. $\cos\left(\frac{2\pi}{3}\right) = \boxed{-\frac{1}{2}}$

Point $(x,y) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\cos\left(\frac{2\pi}{3}\right)$ $\sin\left(\frac{2\pi}{3}\right)$



6. $f(x) = \cos(7x) + \cos(6x) + \sin(3x) + \sin(4x)$

$f'(x) = -7\sin(7x) - 6\sin(6x) + 3\cos(3x) + 4\cos(4x)$

$f'\left(\frac{\pi}{6}\right) = -7\sin\left(\frac{7\pi}{6}\right) - 6\sin\left(\cancel{6}\cdot\frac{\pi}{\cancel{6}}\right) + 3\cos\left(\cancel{3}\frac{\pi}{\cancel{6}_{2}}\right) + 4\cos\left(\cancel{4}^{2}\cdot\frac{\pi}{\cancel{6}_{3}}\right)$

$= -7\underbrace{\cancelto{-\frac{1}{2}}{\sin\left(\frac{7\pi}{6}\right)}}_{\text{See 4. above}} - 6\cancelto{0}{\sin(\pi)} + 3\cancelto{0}{\cos\left(\frac{\pi}{2}\right)} + 4\underbrace{\cancelto{-\frac{1}{2}}{\cos\left(\frac{2\pi}{3}\right)}}_{\text{See 5. above}}$

$= -\frac{7}{2} + 0 + 0 - 2 = \frac{7}{2} - 2 = \frac{7}{2} - \frac{4}{2} = \frac{3}{2}$

7. $f''(x) = 20x^3 + 12x^2 + 4 \qquad f(0) = 8$ and $f(1) = 5$

$$f'(x) = \int f''(x)\,dx = \int 20x^3 + 12x^2 + 4\,dx$$

constant

$$= \frac{\cancel{20}^{\,5}x^4}{\cancel{4}} + \frac{\cancel{12}^{\,4}x^3}{\cancel{3}} + 4x + C$$

note: no f' info given

$$f(x) = \int f'(x)\,dx = \int 5x^4 + 4x^3 + 4x + C$$

New Constant!

$$= \frac{\cancel{5}x^5}{\cancel{5}} + \frac{\cancel{4}x^4}{\cancel{4}} + \frac{\cancel{4}^{\,2}x^2}{\cancel{2}} + Cx + D$$

☆ Use both initial conditions to solve for C and D

$$= x^5 + x^4 + 2x^2 + Cx + D$$

8

$$f(0) = \underbrace{0 + 0 + 0 + 0}_{\text{all } 0} + D \overset{\text{set}}{=} 8 \Rightarrow D = 8$$

$$f(1) = \underbrace{1 + 1 + 2}_{4} + C + 8 \overset{\text{Set}}{=} 5 \qquad C + 12 = 5 \Rightarrow C = -7$$

Finally, $f(x) = \boxed{x^5 + x^4 + 2x^2 - 7x + 8}$

split-split

8 (a) $$\int \frac{7x^{2/5} + 8x^{-4/3} + \frac{1}{x}}{\sqrt{x}}\,dx = \int \frac{7x^{2/5}}{x^{1/2}} + \frac{8x^{-4/3}}{x^{1/2}} + \frac{x^{-1}}{x^{1/2}}\,dx$$

$x^{1/2}$

note: All Algebra Prep

Recall: $\frac{x^a}{x^b} = x^{a-b}$

$$= \int \frac{7x^{4/10}}{x^{5/10}} + \frac{8x^{-8/6}}{x^{3/6}} + \frac{x^{-2/2}}{x^{1/2}}\,dx$$

$$= \int 7x^{\frac{4}{10}-\frac{5}{10}} + 8x^{-\frac{8}{6}-\frac{3}{6}} + x^{-\frac{2}{2}-\frac{1}{2}}\,dx$$

$$= \int 7x^{-1/10} + 8x^{-11/6} + x^{-3/2}\,dx$$

$$= \frac{7x^{9/10}}{9/10} + \frac{8x^{-5/6}}{-5/6} + \frac{x^{-1/2}}{-1/2} + C$$

$\frac{10}{9}$ $\quad -\frac{6}{5}$ $\quad -2$

$$= \boxed{\frac{70}{9}x^{9/10} - \frac{48}{5}x^{-5/6} - 2x^{-1/2} + C}$$

OR $$= \frac{70}{9}x^{9/10} - \frac{48}{5x^{5/6}} - \frac{2}{\sqrt{x}} + C$$

8(b) $\int \left(\sqrt{3}+\frac{1}{x^3}\right)\left(x-\frac{1}{x^{2/7}}\right)dx = \int \sqrt{3}\,x - \frac{\sqrt{3}}{x^{2/7}} + \frac{x}{x^3} - \frac{1}{x^3\cdot x^{2/7}}\,dx$

$x^3\cdot x^{2/7} = x^{21/7}x^{2/7} = x^{23/7}$

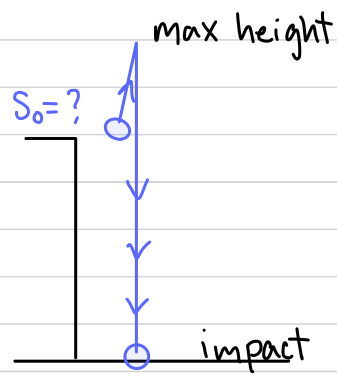
$= \int \sqrt{3}\,x - \sqrt{3}\,x^{-2/7} + x^{-2} - x^{-23/7}\,dx$ prep

$= \frac{\sqrt{3}\,x^2}{2} - \frac{\sqrt{3}\,x^{5/7}}{5/7} + \frac{x^{-1}}{-1} - \frac{x^{-16/7}}{-16/7} + C$

flip

$= \boxed{\frac{\sqrt{3}\,x^2}{2} - \frac{7\sqrt{3}}{5}x^{5/7} - \frac{1}{x} + \frac{7}{16}x^{-16/7} + C}$

9.



Equations of Motion

$a(t) = -32$

$v(t) = -32t + \cancel{v_0}$ 80

$s(t) = -16t^2 + \cancel{v_0}\,t + S_0$ (80, ?)

Given: $v(0) = +80$ ft/sec

$v(t_{impact}) = -112$ ft/sec

Find: $S_0 = ?$

Impact: $v(t_{impact}) = -32t + 80 = -112$ set

$32t = 192$

$t_{impact} = \frac{192}{32} = 6$ seconds

Plug $t_{impact} = 6$ seconds into position and set equal to 0

That is, $s(t_{impact}) = s(6) = 0$

$s(6) = -16(6)^2 + 80(6) + S_0 = 0$ note: S_0 only unknown

$-576 + 480 + S_0 = 0$

$-96 + S_0 = 0$

$\Rightarrow S_0 = 96$ feet

$\begin{array}{r} \overset{3}{36} \\ \underline{16} \\ 216 \\ \underline{360} \\ 576 \end{array}$

Answer: $\boxed{\text{The building is 96 feet tall}}$