Worksheet #1 Answer Key

1. $y = x^{\frac{3}{2}} \cdot \tan x$

$y' = x^{3/2} \cdot \sec^2 x + \tan x \cdot \frac{3}{2} x^{\frac{1}{2}}$

2. $f(x) = x\cos x - \sin x$

$f'(x) = x(-\sin x) + \cancel{\cos x} \cdot (1) - \cancel{\cos x} = -x\sin x$

3. $y = \sec^7 x \overset{\text{prep}}{=} (\sec x)^7$

$y' = 7(\sec x)^6 \cdot (\sec x \cdot \tan x) = 7\sec^7 x \cdot \tan x$

4. $f(x) = \tan\sqrt{1-x^8}$

$f'(x) = \sec^2\sqrt{1-x^8} \cdot \dfrac{1}{\cancel{2}\sqrt{1-x^8}} \cdot (-\overset{-4}{\cancel{8}}x^7) = \dfrac{-4x^7\sec^2\sqrt{1-x}}{\sqrt{1-x}}$

5. $y = \cos\left(\frac{1}{x}\right) = \cos(x^{-1})$

$y' = \cancel{-}\sin\left(\frac{1}{x}\right) \cdot (\cancel{-}x^{-2}) = \dfrac{\sin\left(\frac{1}{x}\right)}{x^2}$

6. $y = \dfrac{1}{\cos x} \overset{\text{prep}}{=} (\cos x)^{-1}$

Chain Rule:

$y' = \cancel{-}(\cos x)^{-2}(\cancel{-}\sin x) = \dfrac{\sin x}{\cos^2 x}$

OR

OR also equals $\dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x} = \sec x \cdot \tan x$

OR Quotient Rule:

$y' = \dfrac{\cancel{\cos x} (0)^{\to 0} - 1(-\sin x)}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x}$ Matches above

OR notice $y = \dfrac{1}{\cos x} = \sec x$ by definition

↳ $y' = \sec x \tan x$ equivalent

7. $f(x) = \sin^6(x^3 - 5x) \overset{\text{prep}}{=} \left(\sin(x^3-5x)\right)^6$

$f'(x) = 6\underbrace{\left(\sin(x^3-5x)\right)^5}_{\sin^5(x^3-5x)} \cdot \cos(x^3-5x) \cdot (3x^2-5)$

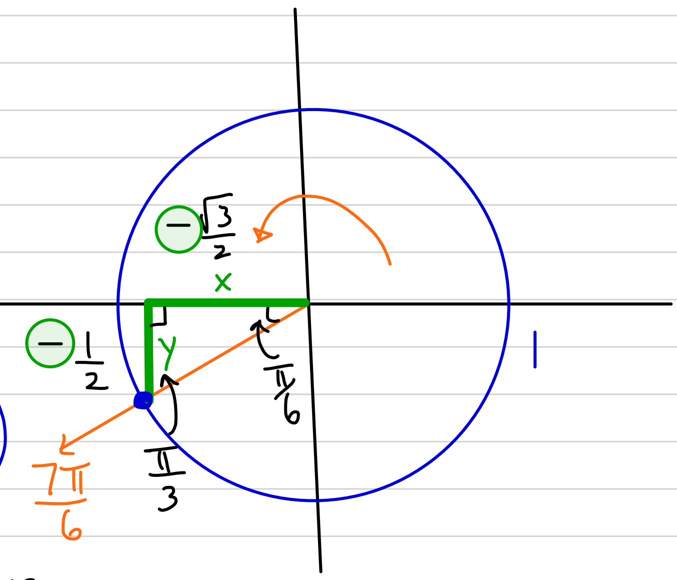
8. $y = \sqrt{\tan\sqrt{x}}$

$$y' = \frac{1}{2\sqrt{\tan\sqrt{x}}} \cdot \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}} = \frac{\sec^2\sqrt{x}}{4\sqrt{\tan\sqrt{x}}\cdot\sqrt{x}}$$

could also combine roots

9. $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$



Point $(x, y) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

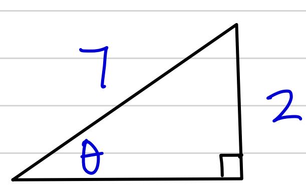
cosine   sine

10. $H(x) = \sin(7x)$ ↪ $H'(x) = \cos(7x)\cdot 7 = 7\cos(7x)$

$$H'\left(\frac{\pi}{6}\right) = 7\underbrace{\cos\left(\frac{7\pi}{6}\right)}_{-\frac{\sqrt{3}}{2}} = 7\left(-\frac{\sqrt{3}}{2}\right) = \frac{-7\sqrt{3}}{2}$$

↳ Note: from #9 above we showed the $\underbrace{\cos}_{\text{x-coordinate}}\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

11. $\sin\theta = \frac{2}{7}$ $\frac{O}{H}$ $\Rightarrow \tan\theta = \frac{2}{\sqrt{45}} = \frac{2}{3\sqrt{5}}$ ($\frac{O}{A}$, OR)



$\sec\theta = \frac{7}{\sqrt{45}} = \frac{7}{3\sqrt{5}}$ ($\frac{H}{A}$, OR)

$\Rightarrow \sqrt{7^2-2^2} = \sqrt{49-4} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$

Note: could also use Identities

12. $f(x) = \tan(2x) + \cos(2x) + 16\cos x$

$f'(x) = \sec^2(2x)\cdot 2 - \sin(2x)\cdot 2 - 16\sin x$

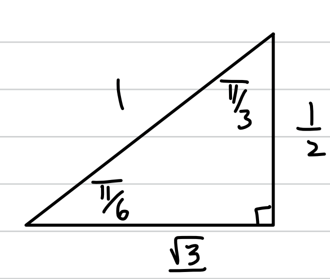
$= 2\sec^2(2x) - 2\sin(2x) - 16\sin x$ ✓ Match

Note: $\sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos(\pi/3)} = \frac{1}{(1/2)} = 2$

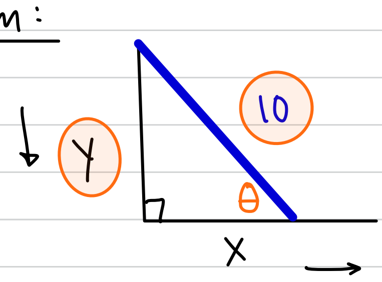
$$f'\left(\frac{\pi}{6}\right) = 2\left[\sec\left(\underbrace{2\left(\frac{\pi}{6}\right)}_{\pi/3}\right)\right]^2 - 2\sin\left(\underbrace{2\left(\frac{\pi}{6}\right)}_{\pi/3}\right) - 16\sin\left(\frac{\pi}{6}\right)$$

$$= 2\left[\underbrace{\sec\left(\frac{\pi}{3}\right)}_{2}\right]^2 - 2\underbrace{\sin\left(\frac{\pi}{3}\right)}_{\frac{\sqrt{3}}{2}} - 16\underbrace{\sin\left(\frac{\pi}{6}\right)}_{\frac{1}{2}}$$

$$= 2\cdot 4 - \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right) - \cancel{16}^{8}\left(\frac{1}{\cancel{2}}\right) = \cancel{8} - \sqrt{3} - \cancel{8} = -\sqrt{3}$$ Match



13. Diagram:



Variables:

Let x = Distance from base of ladder to the wall

y = Distance from top of ladder to the ground

Given $\frac{dy}{dt} = -1$ ft/sec.

Find $\frac{d\theta}{dt} = ?$ when $y = 3$

Equation:

$$\sin\theta = \frac{y}{10}$$

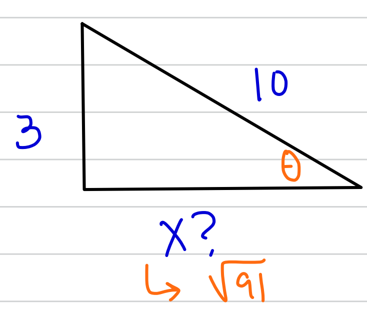
Differentiate:

$$\frac{d}{dt}\left(\sin\theta\right) = \frac{d}{dt}\left(\frac{y}{10}\right)$$

Note: Need x-value to solve for $\cos\theta$ $\frac{A}{H}$

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$$

Extra Solvable Information:



$$x^2 + 3^2 = (10)^2$$

$$x^2 = 100 - 9 = 91$$

$$x = \sqrt{91}$$

Substitute: Key Moment Info

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$$

$$\frac{\sqrt{91}}{10} \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot (-1)$$

Solve:

$$\frac{d\theta}{dt} = -\frac{1}{\cancel{10}}\left(\frac{\cancel{10}}{\sqrt{91}}\right) = -\frac{1}{\sqrt{91}}$$ Rad/sec.

Answer:

The angle between the ladder and the ground is Decreasing at a rate of $\frac{1}{\sqrt{91}}$ Radians per second.

↑ Takes care of minus

14. Implicit Differentiation

$$\frac{d}{dx}\left(y^2 + \cos x\right) = \frac{d}{dx}\left(x \cdot y^3\right)$$

$$2y\frac{dy}{dx} - \sin x = x \cdot 3y^2\frac{dy}{dx} + y^3 \cdot (1)$$ Isolate

$$2y\frac{dy}{dx} - 3xy^2\frac{dy}{dx} = y^3 + \sin x$$ Factor

$$\left(2y - 3xy^2\right)\frac{dy}{dx} = y^3 + \sin x$$ Solve

$$\frac{dy}{dx} = \boxed{\frac{y^3+\sin x}{2y-3xy^2}}$$

15. Challenge:

$$\frac{d}{dx}\cos\left(\frac{\overset{\text{prep } 6x^{-6}}{\frac{6}{x^6}} + \tan(3x)}{\underset{4x^{-1}}{\frac{4}{x}} + \sec x}\right)$$

$$\boxed{= -\sin\left(\frac{\frac{6}{x^6} + \tan(3x)}{\frac{4}{x} + \sec x}\right) \cdot \left[\frac{\left(\frac{4}{x}+\sec x\right)\left(-36x^{-7} + \sec^2(3x) \cdot 3\right) - \left(\frac{6}{x^6} + \tan(3x)\right) \cdot \left(-4x^{-2} + \sec x \tan x\right)}{\left(\frac{4}{x} + \sec x\right)^2}\right]}$$

Chain Rule ↳ Quotient Rule

Do Not Simplify!