



Math 106 Exam 3

April 28, 2023



- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $\ln(e^3)$, $e^{2\ln 3}$ and $4^{\frac{3}{2}}$.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [10 Points] Compute the following Derivative y' where $y = \ln\left(\frac{\sqrt{1+e^x} \cdot \ln(3x)}{e^{\tan x} (4-x^5)^6}\right)$

Do not simplify your final answer here.

2. [10 Points] Compute $\frac{dy}{dx}$ where $y = (\cos x)^x$

3. [12 Points]

(a) Consider $f(x) = \ln(1 + \cos x) - e^{\sin x} - \ln 5$ Compute $f'(0)$.

(b) Consider $f(x) = \cos(\ln(1+x)) - e^x + e^{(e^x)} + \frac{e}{1 + \ln(x+1)}$ Compute $f'(0)$.

4. [12 Points]

(a) Consider $f(x) = e^{8x} + \frac{8}{e^{8x}} + e^{\ln 8} - \frac{8}{x} + \frac{e^x}{e^{8x}}$ Compute $f'(x)$.

(b) Consider $f(x) = 8e^x + \frac{8}{e^x} - \ln(e^8) - \frac{e}{x^8} + (e^{8x}) \cdot (e^x)$ Compute $\int f(x) dx$

5. [26 Points] Compute the following Indefinite Integrals.

(a) $\int \frac{(1 + e^{3x})^2}{e^{3x}} dx$

(b) $\int \frac{e^{3x}}{(1 + e^{3x})^2} dx$

(c) $\int \frac{1}{\sqrt{x} e^{\sqrt{x}}} dx$

(d) $\int \frac{5 - x^8}{x^9} dx$

(e) $\int \frac{x^8}{5 - x^9} dx$

6. [30 Points] Evaluate each of the following Definite Integrals. Simplify.

(a) Show that $\int_0^{\frac{\pi}{3}} \tan x dx = \boxed{\ln 2}$

(b) Show that $\int_0^{\ln 2} \frac{e^{3x}}{\sqrt{8 + e^{3x}}} dx = \boxed{\frac{2}{3}}$

(c) Show that $\int_e^{e^5} \frac{1}{x(3 + \ln x)} dx = \boxed{\ln 2}$

OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute $\lim_{n \rightarrow \infty} \frac{e^{(1+\frac{1}{n})} + e^{(1+\frac{2}{n})} + e^{(1+\frac{3}{n})} + \dots + e^2}{n}$