

Math 106 Midterm Exam #3 April 18, 2018

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $\ln(e^3)$, $e^{2\ln 3}$ and $4^{\frac{3}{2}}$.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [28 Points] Compute each of the following derivatives.

(a) y' where $y = \ln\left(\frac{\ln x \sqrt{1+e^x}}{(4-x^6)^3 e^{-\cos x}}\right)$ Do not simplify your final answer here.

(b) $\frac{d}{dx} (\cos x)^{\sin x}$

(c) $\frac{dy}{dx}$ where $y = \ln(\ln(\ln(e^x))) + \frac{e}{\ln x} + \frac{\ln x}{e} + (\ln x \cdot e^x) + \ln(xe^x)$

(d) $\frac{dy}{dx}$ where $y = e^{\ln(\ln x)} + \ln(e^5) + e^{\ln x} + 5^{\ln e}$

2. [12 Points]

(a) Find the Absolute Maximum and/or Minimum **Values** for the function $f(x) = \frac{x+2}{e^x}$.

(b) At what point on the curve $y = [\ln(x+4)]^2$ is the tangent line horizontal?

3. [8 Points] Sketch the graphs for each of the following functions. For each, state both the Domain and the Range.

(a) $f(x) = e^x$

(b) $f(x) = \ln x$

4. [40 Points] Evaluate each of the following integrals. Simplify.

(a) $\int \frac{1}{x^3 e^{\frac{1}{x^2}}} dx$

(b) $\int_1^{\sqrt{6}} \frac{x}{7-x^2} dx$

(c) $\int_{e^3}^{e^8} \frac{4}{x\sqrt{1+\ln x}} dx$

(d) $\int_{-3}^{-1} \frac{1-x}{x^2} dx$

(e) Show that $\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \tan(3x) dx = \boxed{\frac{\ln 3}{6}}$

(f) Show that $\int_0^{\ln 2} \left(e^x + \frac{1}{e^{2x}}\right)^2 dx = \boxed{\frac{175}{64}}$

5. [6 Points] Find the function $f(x)$ that satisfies $f'(x) = \frac{1}{e^{2x}(1-2e^{-2x})^2}$ and $f(0) = -1$.

6. [6 Points] Consider $y = \ln x$. **Prove** that $\frac{dy}{dx} = \frac{1}{x}$.