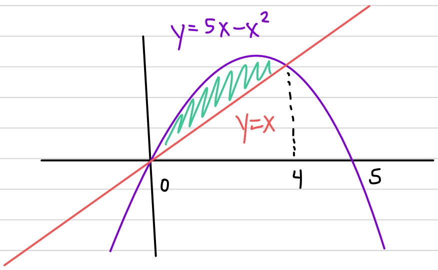
Worksheet 8 Answer Key

1. $y = 5x - x^2$ $y = x$ Intersect?

$= x(5-x) \overset{\text{set}}{=} 0$

Zeroes: $x = 0$, $x = 5$



$$5x - x^2 = x$$

$$x^2 - 5x + x = 0$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0 \quad x = 4$$

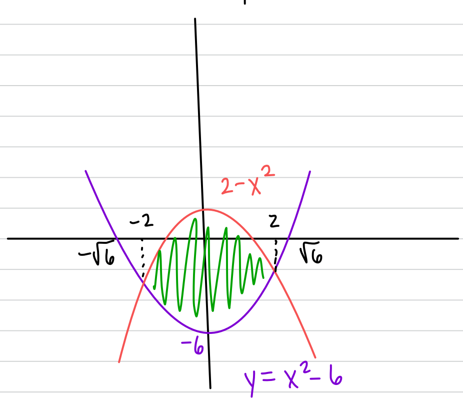
$$\text{Area} = \int_0^4 \text{Top} - \text{Bottom}\, dx = \int_0^4 (5x - x^2) - x\, dx = \int_0^4 4x - x^2\, dx$$

$$= \frac{\cancel{4}^{2}x^2}{\cancel{2}} - \frac{x^3}{3}\Big|_0^4 = 2(4)^2 - \frac{4^3}{3} - (0-0)$$

$$= 32 - \frac{64}{3} = \frac{96}{3} - \frac{64}{3} = \frac{32}{3}$$

+

2. $= 2 - x^2$ $y = x^2 - 6$



Intersect?

$$2 - x^2 = x^2 - 6$$

$$8 = 2x^2$$
$$4 = x^2$$
$\hookrightarrow x = \pm 2$ Symmetry here

$$\text{Area} = \int_{-2}^{2} \text{Top} - \text{Bottom}\, dx = \int_{-2}^{2} 2 - x^2 - (x^2 - 6)\, dx = \int_{-2}^{2} 2 - x^2 - x^2 + 6\, dx$$

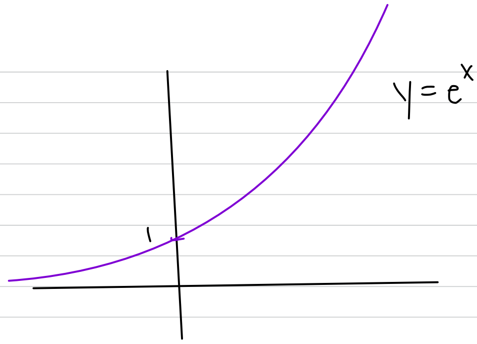
$$= \int_{-2}^{2} 8 - 2x^2\, dx = 8x - \frac{2x^3}{3}\Big|_{-2}^{2}$$

$$= 16 - \frac{2}{3}\cdot 8 - \left(-16 - \frac{2}{3}(-8)\right)$$

$$= 16 - \frac{16}{3} + 16 - \frac{16}{3} = 32 - \frac{32}{3} = \frac{96}{3} - \frac{32}{3} = \frac{64}{3}$$

+

3.



Domain: $\mathbb{R}$ or $(-\infty, \infty)$

Range: $(0, \infty)$

4. $\lim_{x \to \infty} e^x = \infty$ $\qquad$ $\lim_{x \to -\infty} e^x = 0$

Blows Up! $\qquad$ Shrinks to 0

5. $f(x) = e^x$ $\qquad$ $f'(x) = e^x$

6. $f(x) = \frac{1}{e^x} \overset{\text{prep}}{=} e^{-x}$ $\qquad$ $f'(x) = e^{-x} \cdot (-1) = -e^{-x} = \frac{-1}{e^x}$

7. $f(x) = e^{3x}$ $\qquad$ $f'(x) = e^{3x} \cdot 3 = 3e^{3x}$

8. $f(x) = \frac{1}{e^{7x}} \overset{\text{prep}}{=} e^{-7x}$ $\qquad$ $f'(x) = e^{-7x}(-7) = -7e^{-7x} = \frac{-7}{e^{7x}}$

9. $f(x) = e^{\sin x}$ $\qquad$ $f'(x) = \boxed{e^{\sin x} \cdot \cos x}$

10. $f(x) = \sin(e^x)$ $\qquad$ $f'(x) = \cos(e^x) \cdot e^x$

11. $f(x) = e^{\sqrt{x}}$ $\qquad$ $f'(x) = \boxed{e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}}$

12. $f(x) = \sqrt{e^x}$ $\qquad$ $f'(x) = \boxed{\frac{1}{2\sqrt{e^x}} \cdot e^x} \overset{\text{OR}}{=} \frac{\sqrt{e^x}}{2}$

OR prep $f(x) = \sqrt{e^x} = (e^x)^{1/2} = e^{x/2} \Rightarrow f'(x) = e^{x/2} \cdot \frac{1}{2} = \frac{1}{2}\sqrt{e^x}$ $\qquad$ Match!

13. $f(x) = e^{(e^x)}$ $\qquad$ $f'(x) = \boxed{e^{(e^x)} \; e^x}$

14. $f(x) = e$ constant $\qquad$ $f'(x) = 0$

15. $f(x) = \frac{e}{x} = ex^{-1}$ $\qquad$ $f'(x) = e(-x^{-2}) = \frac{-e}{x^2}$

16. $f(x) = \frac{x}{e} = \frac{1}{e} \cdot x$ $\qquad$ $f'(x) = \frac{1}{e}$

17. $f(x) = e^5$ constant $f'(x) = 0$

18. $f(x) = ex$ $f'(x) = e$

19. $f(x) = \frac{1}{ex} = \frac{1}{e} \cdot x^{-1}$ $f'(x) = \frac{1}{e}\left(-x^{-2}\right) = \frac{-1}{ex^2}$

20. $f(x) = x^e$ $f'(x) = e\,x^{e-1}$ power rule, e = constant

21. $f(x) = \frac{1}{x^e} = x^{-e}$ $f'(x) = -e\,x^{-e-1}$ NOT exponential rule

22. $f(x) = \frac{e^{-2x}}{1+e^x}$ $f'(x) = \frac{(1+e^x)\cdot e^{-2x}(-2) - e^{-2x}\cdot e^x}{(1+e^x)^2} = \frac{-2e^{-2x} - 2e^{-x} - e^{-x}}{(1+e^x)^2}$

$$= \boxed{\frac{-2e^{-2x} - 3e^{-x}}{(1+e^x)^2}}$$

23. $f(x) = \left(e^{2x} - e^{-3x}\right)^7$ $f'(x) = 7\left(e^{2x} - e^{-3x}\right)^6 \cdot \left(e^{2x}\cdot 2 - e^{-3x}\cdot(-3)\right)$

$$= \boxed{7\left(e^{2x} - e^{-3x}\right)^6 \left(2e^{2x} + 3e^{-3x}\right)}$$

24. $e^{xy} = 2 + \sin x$ Implicit Differentiation

$$\frac{d}{dx}\left(e^{xy}\right) = \frac{d}{dx}\left(2 + \sin x\right)$$

$$e^{xy} \cdot \left(x \cdot \frac{dy}{dx} + y(1)\right) = \cos x$$

$$xe^{xy}\frac{dy}{dx} + ye^{xy} = \cos x$$

$$xe^{xy}\frac{dy}{dx} = \cos x - ye^{xy}$$

Solve: $\frac{dy}{dx} = \boxed{\frac{\cos x - ye^{xy}}{xe^{xy}}}$

25. $\int e^x\sqrt{1-e^x}\,dx = -\int \sqrt{u}\,du = -\int u^{1/2}\,du = -\frac{u^{3/2}}{3/2} + C = \boxed{-\frac{2}{3}\left(1-e^x\right)^{3/2} + C}$

$$\boxed{\begin{aligned} u &= 1 - e^x \\ du &= -e^x\,dx \\ -du &= e^x\,dx \end{aligned}}$$