Math 106, Spring 2022

Worksheet 7, Tuesday, April 5th, 2022

Limit Definition of the Definite Integral

- 1. Show that $\int_{1}^{5} 7 x x^{2} dx = \boxed{-\frac{76}{3}}$ using two different methods:
- (a) Fundamental Theorem of Calculus
- (b) Limit Definition of the Definite Integral.

Differentiation

2. Compute g'(x) where $g(x) = \int_x^2 \frac{\cos t}{5 + \cos t} dt$

Integration Evaluate each of the following integrals:

3.
$$\int_{0}^{4} \frac{1-x}{\sqrt{x}} dx$$
4.
$$\int x^{4} (2-3x^{5})^{6} dx$$
5.
$$\int_{9}^{64} \frac{5}{\sqrt{x}\sqrt{1+\sqrt{x}}} dx$$
6.
$$\int_{0}^{\frac{\pi}{6}} \frac{\cos x}{(1+6\sin x)^{2}} dx$$
7.
$$\int \frac{5}{x^{2} \left(5+\frac{3}{x}\right)^{\frac{3}{5}}} dx$$
8.
$$\int x(x-2)^{\frac{3}{4}} dx$$

Initial Valued Problem

9. Find the function f(x) that satisfies $f'(x) = \frac{\sec^2 x}{\sqrt{3 + \tan x}}$ and $f\left(\frac{\pi}{4}\right) = -5$

Displacement-Total Distance

10. Consider an object moving on the number line such that its Velocity at time t is $v(t) = \sin t$ feet per second. Also assume that s(0) = 2 feet, where as usual s(t) is the Position of the object at time t.

(a) Compute the Acceleration function a(t) and the Position function s(t).

(b) Draw the graph of v(t) for $0 \le t \le 2\pi$, and explain why the object is *not* always moving to the right.

(c) Draw the graph of |v(t)| for $0 \le t \le 2\pi$.

(d) Set-Up, but do not compute, the Definite Integral that represents each of the **Displace**ment and Total Distance traveled for $0 \le t \le 2\pi$. Think about how to compute these definite integrals.

Turn in your own solutions.