Worksheet 6 Answer Key

1. $\int x^7 \cdot (4-x^8)^6 \, dx = -\frac{1}{8}\int u^6 \, du = -\frac{1}{8} \cdot \frac{u^7}{7} + C = -\frac{u^7}{56} + C$

$= -\frac{(4-x^8)^7}{56} + C$

$$\boxed{\begin{aligned} u &= 4-x^8 \\ du &= -8x^7 \, dx \\ -\tfrac{1}{8} \, du &= x^7 \, dx \end{aligned}}$$

2. $\int \frac{\cos\sqrt{x}}{\sqrt{x}} \, dx = 2\int \cos u \, du = 2\sin u + C = 2\sin\sqrt{x} + C$

$$\boxed{\begin{aligned} u &= \sqrt{x} \\ du &= \frac{1}{2\sqrt{x}} \, dx \\ 2 \, du &= \frac{1}{\sqrt{x}} \, dx \end{aligned}}$$

3. $\int \cos x \cdot \sin^6 x \, dx = \int \cos x \, (\sin x)^6 \, dx = \int u^6 \, du = \frac{u^7}{7} + C = \frac{\sin^7 x}{7} + C$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x \, dx \end{aligned}}$$

4. $\int x^5 \sqrt{x^6+7} \, dx = \frac{1}{6}\int \sqrt{u} \, du = \frac{1}{6}\int u^{\frac{1}{2}} \, du = \frac{1}{6} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

prep

$\frac{1}{\cancel{6}_3} \cdot \frac{\cancel{2}}{3}$

$= \frac{1}{9} u^{\frac{3}{2}} + C = \frac{1}{9}(x^6+7)^{\frac{3}{2}} + C$

$$\boxed{\begin{aligned} u &= x^6+7 \\ du &= 6x^5 \, dx \\ \tfrac{1}{6} \, du &= x^5 \, dx \end{aligned}}$$

5. $\int \frac{x}{(x^2+1)^9} \, dx = \frac{1}{2}\int \frac{1}{u^9} \, du = \frac{1}{2}\int u^{-9} \, du = \frac{1}{2} \frac{u^{-8}}{-8} + C$

prep

$= -\frac{1}{16u^8} + C = -\frac{1}{16(x^2+1)^8} + C$

$$\boxed{\begin{aligned} u &= x^2+1 \\ du &= 2x \, dx \\ \tfrac{1}{2} \, du &= x \, dx \end{aligned}}$$

6. $\int \frac{\sin x}{\cos^5 x}\,dx = \int \frac{\sin x}{(\cos x)^5}\,dx = -\int \frac{1}{u^5}\,du = -\int u^{-5}\,du = \cancel{-}\frac{u^{-4}}{\cancel{-}4} + C$

$= \frac{1}{4u^4} + C = \frac{1}{4\cos^4 x} + C$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

7. $\int \sec^2 x \cdot \tan^5 x\,dx = \int \sec^2 x \cdot (\tan x)^5\,dx = \int u^5\,du = \frac{u^6}{6} + C = \frac{\tan^6 x}{6} + C$

$u = \tan x$

$du = \sec^2 x\,dx$

8. $\int \frac{\left(9+\frac{1}{x}\right)^3}{x^2}\,dx = -\int u^3\,du = -\frac{u^4}{4} + C = -\frac{\left(9+\frac{1}{x}\right)^4}{4} + C$

$u = 9 + \frac{1}{x}$

$du = -\frac{1}{x^2}\,dx$

$-du = \frac{1}{x^2}\,dx$

9. $\int_1^4 \frac{1}{\sqrt{x}\,(1+\sqrt{x})^3}\,dx = 2\int_2^3 \frac{1}{u^3}\,du \overset{\text{prep}}{=} 2\int_2^3 u^{-3}\,du = \cancel{2}\frac{u^{-2}}{-\cancel{2}}\Big|_2^3 = -\frac{1}{u^2}\Big|_2^3$

$u = 1 + \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$x = 1 \Rightarrow u = 1 + \cancel{\sqrt{1}}^{\,1} = 2$

$x = 4 \Rightarrow u = 1 + \cancel{\sqrt{4}}^{\,2} = 3$

$= -\frac{1}{3^2} - \left(-\frac{1}{2^2}\right)$

$= -\frac{1}{9} + \frac{1}{4} = -\frac{4}{36} + \frac{9}{36} = \frac{5}{36}$

10. $f(x) = \int f'(x)\,dx = \int x^2 \sin(x^3)\,dx = \frac{1}{3}\int \sin u\,du = \frac{1}{3}(-\cos u) + C$

$= -\frac{1}{3}\cos(x^3) + C$

$u = x^3$

$du = 3x^2\,dx$

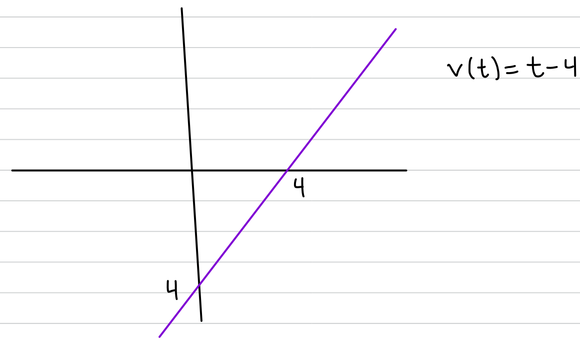
$\frac{1}{3}\,du = x^2\,dx$

Test

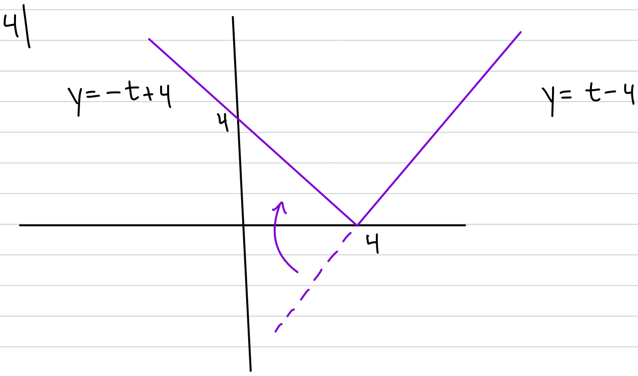
$f(0) = -\frac{1}{3}\cancel{\cos 0}^{\,1} + C = -\frac{1}{3} + C \overset{\text{set}}{=} 3 \Rightarrow C = 3 + \frac{1}{3} = \frac{10}{3}$ ($\frac{9}{3}$)

Finally, $f(x) = -\frac{1}{3}\cos(x^3) + \frac{10}{3}$

11(a) $v(t) = t-4$



11(b) $|v(t)| = |t-4|$



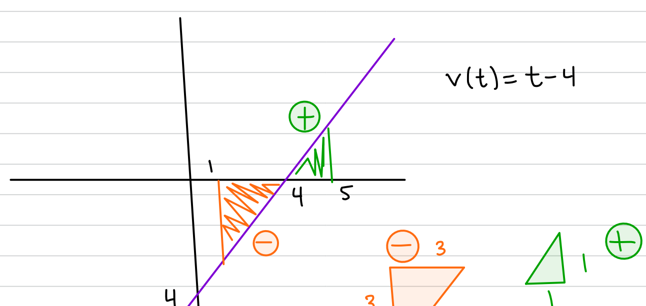
11(c) $$|v(t)| = \begin{cases} t-4 & \text{if } t \geq 4 \\ -(t-4) & \text{if } t < 4 \end{cases} = \begin{cases} t-4 & \text{if } t \geq 4 \\ -t+4 & \text{if } t < 4 \end{cases}$$

11(d) Dispacement $= \int_1^5 v(t)\,dt = \int_1^5 t-4\,dt = \frac{t^2}{2} - 4t \Big|_1^5$

$$= \frac{25}{2} - 20 - \left(\frac{1}{2} - 4\right) = \frac{25}{2} - 20 - \frac{1}{2} + 4$$

$$= \frac{\cancel{24}^{12}}{\cancel{2}} - 16 = 12 - 16 = -4$$

Area Interpretations: Definite Integral = Area Above x-axis — Area Below x-axis



$-\frac{1}{2} \cdot 3 \cdot 3$ $\qquad \frac{1}{2} \cdot 1 \cdot 1$

$-\frac{9}{2} + \frac{1}{2} = -\frac{8}{2} = -4$ Match

Assign Manually

11(e) Total Distance $= \int_1^5 |v(t)|\,dt = \int_1^4 |v(t)|\,dt + \int_4^5 |v(t)|\,dt$

$$= \int_1^4 -t+4\,dt + \int_4^5 t-4\,dt$$

$$= -\frac{t^2}{2} + 4t \Big|_1^4 + \frac{t^2}{2} - 4t \Big|_4^5$$

$$= \cancel{\frac{-16}{2}}^{-8} + 16 - \left(-\frac{1}{2} + 4\right) + \frac{25}{2} - 20 - \left(\cancel{\frac{16}{2}}^{8} - 16\right)$$

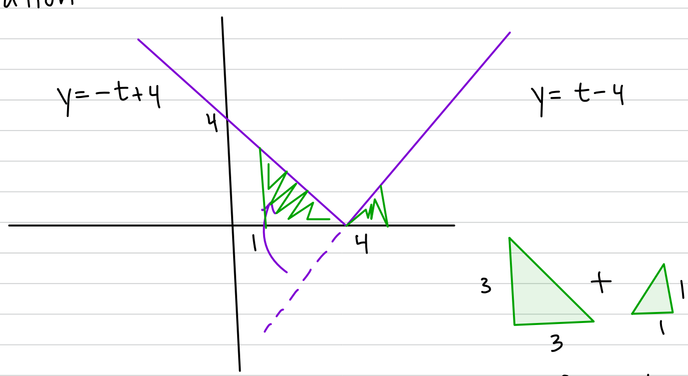
$$= \underbrace{-8+16}_{8} + \frac{1}{2} - 4 + \frac{25}{2} - 20 - \underbrace{8+16}_{8}$$

$\frac{26}{2} = 13$ $\quad -24$

$$= 16 + 13 - 24 = 29 - 24 = 5$$

should be +

Area Interpretation



$+\frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5$ Match

13. $f'(x) = \frac{d}{dx}\int_5^x \frac{1}{t+7}\,dt = \frac{1}{x+7}$

14. $f'(x) = \frac{d}{dx}\int_x^9 \sqrt{t^2+3}\,dt = \frac{d}{dx}\left(-\int_9^x \sqrt{t^2+3}\,dt\right) = -\sqrt{x^2+3}$

15. First $g'(x) = \frac{d}{dx}\int_x^7 \sqrt{1+\cos t}\,dt = \frac{d}{dx}\left(-\int_7^x \sqrt{1+\cos t}\,dt\right) = -\sqrt{1+\cos x}$

$$g''(x) = \cancel{-}\frac{1}{2\sqrt{1+\cos x}}\left(\cancel{-}\sin x\right) = \frac{\sin x}{2\sqrt{1+\cos x}}$$