

Worksheet 3, Tuesday, March 1st, 2022

1. Differentiate each of the following functions. Do NOT simplify your answers.

$$(a) f(x) = \sin \left(\cos^6 \left(\frac{9}{x^8} \right) \right) \stackrel{\text{prep}}{=} \sin \left[\left(\cos \left(\frac{9}{x^8} \right) \right)^6 \right]$$

$$(b) y = \tan \left(\frac{9}{\sin x} \right) \stackrel{\text{prep}}{=} \tan \left(9 (\sin x)^{-1} \right)$$

$$(c) g(t) = \frac{8 + \sec^2(7t)}{9 + \cos t} \stackrel{\text{prep}}{=} \frac{8 + (\sec(7t))^2}{9 + \cos t}$$

2. Consider the equation $f(x) = \tan x \cdot \sin x - \frac{\sin(2x)}{2}$. Compute $f' \left(\frac{\pi}{6} \right)$. Simplify.

3. Show that the equation of the Tangent Line for $\sin(x + y) = 2x - 2y$ at the point (π, π) is given by $y = \frac{x}{3} + \frac{2\pi}{3}$

4. Sally is standing 10 meters from a railroad track as a train goes past. She is waiting for her friend Bob, who is on the train looking at Sally through the window. The train misses its stop. At the moment when the distance between Sally and Bob is 13 meters, Bob's head is rotating at a rate of 2 radians per second to keep her in sight. How fast is the train going at that moment?

5. (a) Find a function f such that $f''(x) = 20x^3 + 12x^2 + 4$ and $f(0) = 8$ and $f(1) = 5$.

(b) A particle is moving with acceleration given by $a(t) = 3 \cos t - 2 \sin t$, with initial position $s(0) = 0$ and initial velocity $v(0) = 4$. Find the position of the particle at time t .

6. Find the Most General Antiderivative for each of the following:

$$(a) \int \frac{7x^{\frac{2}{5}} + 8x^{-\frac{4}{3}} + \frac{1}{x}}{\sqrt{x}} dx$$

$$(b) \int \left(\sqrt{3} + \frac{1}{x^3} \right) \left(x - \frac{1}{x^{\frac{2}{7}}} \right) dx$$

7. Mark throws a baseball upward from the top of a bridge. The initial *speed* of the ball is 80 feet per second. It hits the ground with a *speed* of 112 feet per second. How tall is the bridge?

Hint: Use $a(t) = -32$ feet per second squared as acceleration due to gravity on the falling body.

Turn in your own solutions.