Math 106 Midterm Exam #1 March 4, 2022

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers.

1. [22 Points] Differentiate each of the following functions. Do **NOT** simplify your answers.

(a) $f(x) = \cos\pi + \sqrt{\cos\sqrt{x}}$

(b) $f(x) = \cos(\sin x)$

(c) $f(x) = \cos x \cdot \sin x$

(d) $f(x) = \cos^5\left(\frac{7}{x^6}\right) \overset{\text{prep}}{=} \left(\cos\left(\frac{7}{x^6}\right)\right)^5$

(e) $f(x) = \left(\frac{\cos(7x)}{\tan(3x)}\right)^{\frac{7}{8}}$

2. [20 Points] Derivatives

(a) Let $f(x) = \tan(2x) \cdot \sin(2x)$. Show that $f'\left(\frac{\pi}{6}\right) = \boxed{5\sqrt{3}}$.

(b) Let $H(x) = \cos(4x)$. Compute both $H'\left(\frac{\pi}{3}\right)$ and $H'\left(\frac{\pi}{8}\right)$.

3. [23 Points] Compute the following **Most General Antiderivatives.**

(a) $\int \frac{5}{6}x^5 + x^{\frac{5}{6}} + \frac{1}{x^{\frac{5}{6}}} + \frac{6}{5} + \frac{5}{6x^6} - \frac{6}{x^5}\, dx$

(b) $\int \sec^2 x - 8\cos x + \sin x + \frac{\sec x \tan x}{7}\, dx$

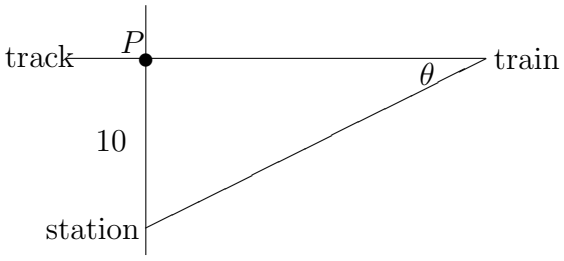
(c) $\int \left(x^2 + \frac{1}{x^2}\right)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)\, dx$

(d) $\int \frac{x^2 + \sqrt{x}}{x^{\frac{3}{7}}}\, dx$

4. [15 Points] Consider a point P on a train track. Suppose a train depot station is 10 feet directly south from this point P. The train is travelling east at 6 feet per second. Consider the angle as shown in the diagram. How fast is this angle changing when 2 seconds has passed since the train passed point P.

• Diagram

The picture at arbitrary time t is:



5. [8 Points] Consider a function f such that $f'(x) = 2\sin x - 5\cos x$ and $f(\pi) = -6$ Compute $f(x)$.

6. [12 Points] For the following, use $a(t) = -32$ feet per second squared as acceleration due to gravity on the falling body.

A ball is thrown upwards from the top of a building that is 48 feet tall with an initial *speed* of 32 feet per second.

• **When** does the ball reach its Maximum Height?

• **What** is the Maximum Height reached by the ball?

• **What** is the velocity of the ball at time $t = 2$ seconds?