Math 106 Final Examination May 8, 2017

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [24 Points] Compute each of the following derivatives.

(a) $g'(x)$, where $g(x) = (\ln x)^{\ln x}$

(b) $\dfrac{d}{dx} \ln\left(\dfrac{3\sqrt{1+\sec^2 x}}{(8-x^3)^5\, e^{-\sin x}}\right)$ Do not simplify the final answer here.

(c) $\dfrac{dy}{dx}$, if $\tan(xy) + \ln\left(e^{-9}\right) = \sin^2 x + (\ln(e+5))x - y^3$

(d) $f'(e)$, where $f(x) = \sqrt{\ln x} - \ln\sqrt{x}$. Simplify.

2. [15 Points] Compute each of the following derivatives. Simplify.

(a) $f'\left(\dfrac{\pi}{12}\right)$ where $f(x) = 2\sin^3(4x) + \sec(4x) - 8\sin(2x)$

(b) $f'\left(\dfrac{\pi}{4}\right)$ where $f(x) = \cos(2x) + \dfrac{1}{\tan^2 x} + \sin\left(x - \dfrac{\pi}{4}\right)$

3. [56 Points] Compute each of the following integrals.

(a) $\displaystyle\int \frac{\left(1 - x^{\frac{3}{4}}\right)\left(x^{\frac{5}{4}} - x^3\right)}{x^3}\, dx$ (b) Show that $\displaystyle\int_{-1}^{2} \frac{x^3}{x^2-5}\, dx = \frac{3}{2} - \ln(32)$ (c) $\displaystyle\int \frac{\left(1+e^{3x}\right)^2}{e^{3x}}\, dx$

(d) $\displaystyle\int \frac{e^{3x}}{\left(1+e^{3x}\right)^2}\, dx$ (e) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x\, \tan^3 x\, dx$

(f) $\displaystyle\int \frac{1}{5\sqrt{x}\, e^{\sqrt{x}}}\, dx$ (g) $\displaystyle\int_2^6 \frac{1}{x^2}\cos\left(\frac{\pi}{x}\right)\, dx$ (h) $\displaystyle\int_1^{e^3} \frac{\sqrt{1+\ln x}}{x}\, dx$

4. [20 Points] Consider the function given by

$f(x) = e^{7x} + \dfrac{1}{e^{7x}} + e^7 + \dfrac{7}{e^x} + \dfrac{7}{e^7} + \dfrac{e}{x^7} + x^e + \dfrac{1}{x^e} + \dfrac{x}{e} + \dfrac{e}{x} + ex + \dfrac{1}{ex}$

(a) Compute the **derivative**, $f'(x)$. (b) Compute the **antiderivative**, $\displaystyle\int f(x)\, dx$.

5. [15 Points] Compute $\displaystyle\int_{-1}^{1} x^3\, dx$ using each of the following **two** different methods:

(a) Fundamental Theorem of Calculus (b) The limit definition of the definite integral.

Recall

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2 \quad\text{and}\quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad\text{and}\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad\text{and}\quad \sum_{i=1}^{n} 1 = n$$

6. [10 Points] Sally is at a train station, standing 10 meters from the railroad track as a train goes past. She is waiting for her friend Bob, who is on the train looking at Sally through the window. The train misses its stop. At the moment when the distance between Sally and Bob is 13 meters, Bob's head is rotating at a rate of 2 radians per second to keep her in sight. How fast is the train going at that moment?

7. [10 Points]

(a) Compute $\displaystyle\int \tan x \, dx$.

(b) Consider $f(x)$ with $f''(x) = \sec^2 x$ and $f'\left(\frac{\pi}{3}\right) = 2\sqrt{3}$ and $f(0) = 7$. Find $f(x)$.

8. [20 Points]

(a) Consider the region bounded by $y = e^x + 1$, $y = 4$, and $x = 0$. Sketch the bounded region.

(b) Compute the area of the bounded region in (a). Simplify.

(c) Compute the volume of the three-dimensional solid obtained by rotating the region in (a) about the $\boldsymbol{x}$**-axis**. Sketch the solid, along with one of the approximating washers. Simplify.

(d) Consider a **different** region bounded by $y = \sin x$, $y = 1$, $x = 0$ and $x = \frac{\pi}{2}$. Sketch the bounded region.

(e) **Set-Up** but **DO NOT EVALUATE** the integral to compute the volume of the three-dimensional solid obtained by rotating the region in (d) about the horizontal line $\boldsymbol{y = -1}$. Sketch the solid, along with one of the approximating washers.

(f) Consider a **different** region bounded by $y = e^x$, $y = \ln x$, $x = 1$, and $x = 3$. Sketch the bounded region.

(g) **Set-Up** but **DO NOT EVALUATE** the integral to compute the volume of the three-dimensional solid obtained by rotating the region in (f) about the $\boldsymbol{x}$**-axis**. Sketch the solid, along with one of the approximating washers.

9. [10 Points] Consider an object moving on a number line such that its velocity at time t seconds is given by $\boldsymbol{v(t) = t^2 - 3t}$ feet per second.

(a) Sketch $|v(t)|$. (b) Compute the **total distance** travelled for $0 \leq t \leq 4$. Simplify.

10. [10 Points] Mark throws a baseball upward from the top of a building. This initial *speed* of the ball is 20 feet per second. On its way down, the ball hits the ground with a *speed* of 44 feet per second. How tall is the building? (Hint: use acceleration $a(t) = -32$ feet per second squared.)

11. [10 Points] A population of bacteria was growing exponentially. Initially there were 4 cells. After 1 hour there were 12 cells. How many cells were there after 3 hours? When were there 324 cells?