

### Extra Riemann Sum, Definite Integral Example

Evaluate  $\int_{-1}^2 x^2 - 3x + 6 \, dx$  using Riemann Sums and the limit definition of the definite integral.

Here  $f(x) = x^2 - 3x + 6$ ,  $a = -1$ ,  $b = 2$ ,  $\Delta x = \frac{2 - (-1)}{n} = \frac{b - a}{n} = \frac{3}{n}$   
and  $x_i = a + x_i = -1 + i \left(\frac{3}{n}\right) = -1 + \frac{3i}{n}$ .

$$\begin{aligned}\int_{-1}^2 x^2 - 3x + 6 \, dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(-1 + \frac{3i}{n}\right) \left(\frac{3}{n}\right) \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left(-1 + \frac{3i}{n}\right)^2 - 3\left(-1 + \frac{3i}{n}\right) + 6 \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left(1 - \frac{6i}{n} + \frac{9i^2}{n^2} + 3 - \frac{9i}{n} + 6\right) \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left(\frac{9i^2}{n^2} - \frac{15i}{n} + 10\right) \\ &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \frac{9i^2}{n^2} - \frac{3}{n} \sum_{i=1}^n \frac{15i}{n} + \frac{3}{n} \sum_{i=1}^n 10 \\ &= \lim_{n \rightarrow \infty} \frac{27}{n^3} \sum_{i=1}^n i^2 - \frac{45}{n^2} \sum_{i=1}^n i + \frac{30}{n} \sum_{i=1}^n 1 \\ &= \lim_{n \rightarrow \infty} \frac{27}{n^3} \left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{45}{n^2} \left(\frac{n(n+1)}{2}\right) + \frac{30}{n}(n) \\ &= \lim_{n \rightarrow \infty} \frac{27}{6} \left(\frac{n}{n}\right) \left(\frac{n+1}{n}\right) \left(\frac{2n+1}{n}\right) - \frac{45}{2} \left(\frac{n}{n}\right) \left(\frac{n+1}{n}\right) + 30 \\ &= \lim_{n \rightarrow \infty} \frac{27}{6} (1) \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) - \frac{45}{2} (1) \left(1 + \frac{1}{n}\right) + 30 \\ &= \frac{27}{6} (1)(1)(2) - \frac{45}{2} (1)(1) + 30 = \frac{54}{6} - \frac{45}{2} + 30 \\ &= \frac{54}{6} - \frac{135}{6} + \frac{180}{6} = \frac{99}{6} = \boxed{\frac{33}{2}}\end{aligned}$$