

$$\text{Intersect: } e^x + 1 = 4$$

$$e^x = 3$$

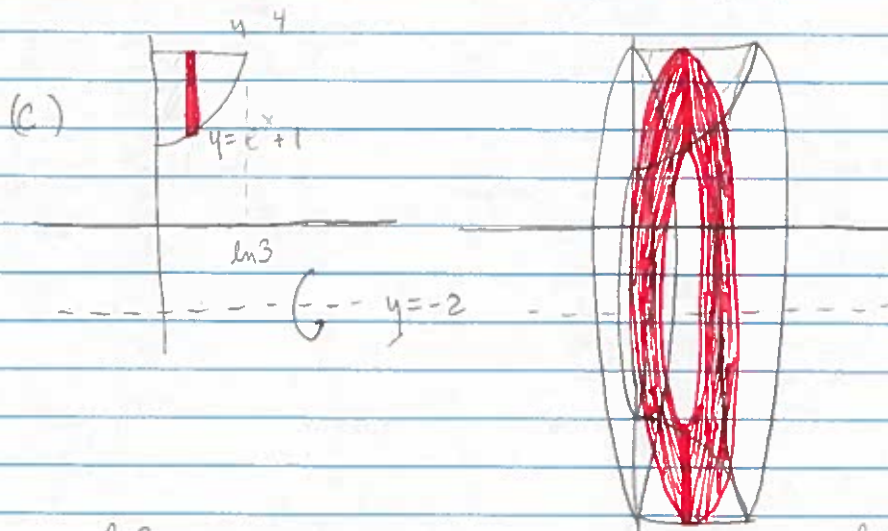
$$\ln e^x = \ln 3$$

$$x = \ln 3$$

$$(b) \text{ Area: } \int_0^{\ln 3} 4 - (e^x + 1) dx = \int_0^{\ln 3} 4 - e^x - 1 dx = \int_0^{\ln 3} 3 - e^x dx$$

$$= 3x - e^x \Big|_0^{\ln 3} = 3\ln 3 - e^{\ln 3} - (0 - e^0)$$

$$= 3\ln 3 - 3 + 1 = \boxed{3\ln 3 - 2}$$



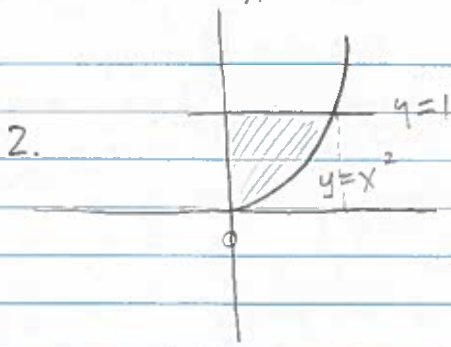
$$V = \pi \int_0^{\ln 3} (\text{outer radius})^2 - (\text{inner radius})^2 dx = \pi \int_0^{\ln 3} (6)^2 - (e^x + 3)^2 dx$$

$$= \pi \int_0^{\ln 3} 36 - (e^{2x} + 6e^x + 9) dx = \pi \int_0^{\ln 3} 36 - e^{2x} - 6e^x - 9 dx$$

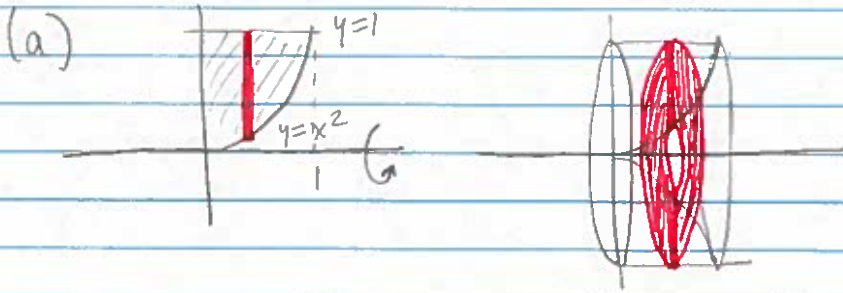
$$= \pi \int_0^{\ln 3} 27 - e^{2x} - 6e^x dx = \pi \left[27x - \frac{e^{2x}}{2} - 6e^x \right]_0^{\ln 3}$$

$$= \pi \left[\left(27\ln 3 - \frac{e^{2\ln 3}}{2} - 6e^{\ln 3} \right) - \left(0 - \frac{e^{0}}{2} - 6e^0 \right) \right] = \pi \boxed{27\ln 3 - 16}$$

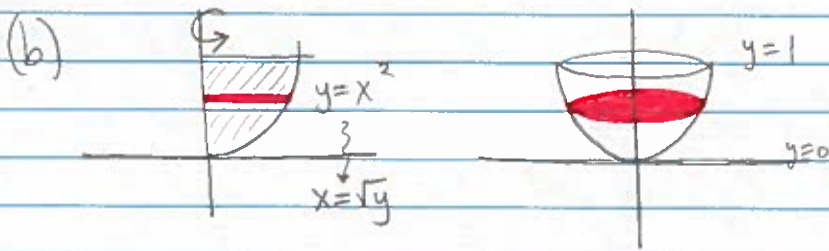
$x \geq 0$ here



Intersect? $x^2 = 1 \Rightarrow x = \pm 1$ Keeping $x \geq 0$
 $\Rightarrow x = 1$.



$$V = \pi \int_0^1 (\text{outer radius})^2 - (\text{inner radius})^2 dx = \pi \int_0^1 1^2 - (x^2)^2 dx = \pi \int_0^1 1 - x^4 dx$$
$$= \pi \left[x - \frac{x^5}{5} \right]_0^1 = \pi \left[1 - \frac{1}{5} - (0 - 0) \right] = \boxed{\frac{4}{5} \pi}$$

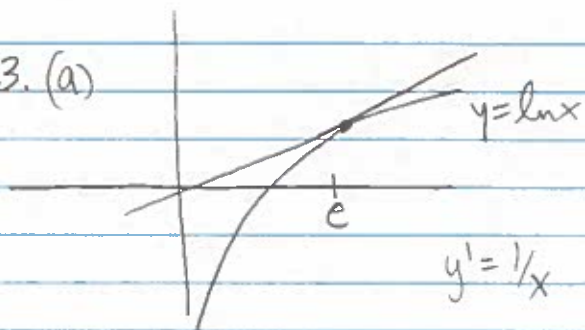


$$V = \pi \int_0^1 (\text{radius})^2 dy = \pi \int_0^1 (\sqrt{y})^2 dy = \pi \int_0^1 y dy = \pi \left. \frac{y^2}{2} \right|_0^1$$

change in y

$$= \pi \left[\frac{1}{2} - 0 \right] = \boxed{\frac{\pi}{2}}$$

3. (a)



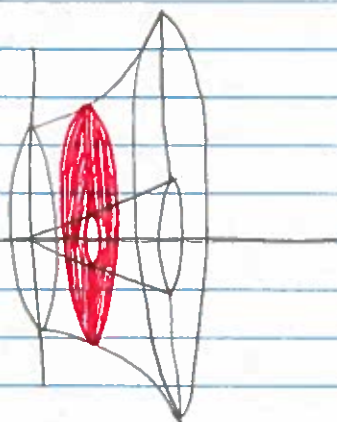
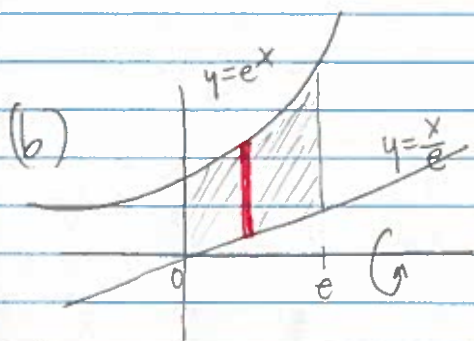
Point $(e, \ln e) = (e, 1)$

Slope $y'(e) = \frac{1}{e}$

Point Slope Form $y - 1 = \frac{1}{e}(x - e)$

$y = \frac{1}{e}x + 1 - 1$

$y = \frac{1}{e}x$

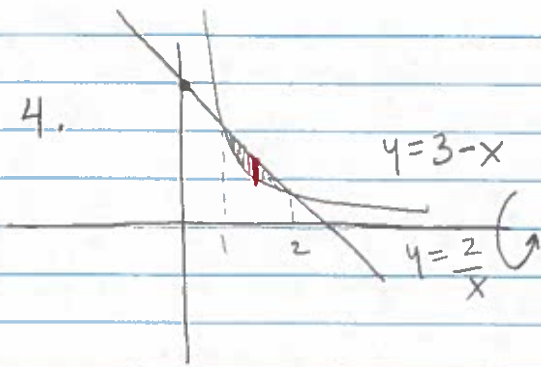


$$V = \pi \int_0^e (\text{outer radius})^2 - (\text{inner radius})^2 dx = \pi \int_0^e (e^x)^2 - \left(\frac{x}{e}\right)^2 dx$$

$$= \pi \int_0^e e^{2x} - \frac{x^2}{e^2} dx = \pi \left[\frac{e^{2x}}{2} - \frac{1}{e^2} \left(\frac{x^3}{3} \right) \right] \Big|_0^e$$

$$= \pi \left[\left(\frac{e^{2e}}{2} - \frac{e^3}{3e^2} \right) - \left(\frac{e^{2 \cdot 0}}{2} - 0 \right) \right] = \pi \left[\frac{e^{2e}}{2} - \frac{e}{3} - \frac{1}{2} \right]$$

4.



Intersect?

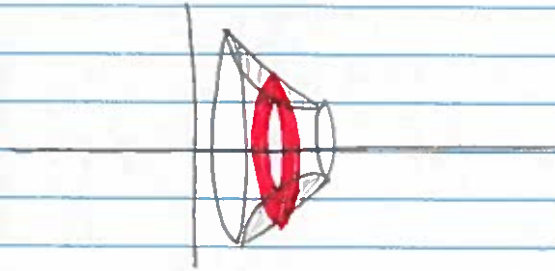
$$3 - x = \frac{2}{x}$$

$$3x - x^2 = 2$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x=1, x=2$$



$$V = \pi \int_1^2 (\text{outer radius})^2 - (\text{inner radius})^2 dx = \pi \int_1^2 (3-x)^2 - \left(\frac{2}{x}\right)^2 dx$$

$$= \pi \int_1^2 9 - 6x + x^2 - \frac{4}{x^2} dx = \pi \left[9x - 3x^2 + \frac{x^3}{3} + \frac{4}{x} \right] \Big|_1^2$$

$$= \pi \left[\underbrace{(18 - 12 + \frac{8}{3} + 2)}_8 - \underbrace{(9 - 3 + \frac{1}{3} + 4)}_{10} \right] = \pi \left[8 + \frac{8}{3} - 10 - \frac{1}{3} \right]$$

$$= \pi \left[\frac{7}{3} - 2 \right] = \pi \left[\frac{7}{3} - \frac{6}{3} \right] = \boxed{\frac{\pi}{3}}$$