Math 106 WS#10 Spring 2017

1. $g(x) = \int_x^7 \sqrt{e^t+3}\, dt = -\int_7^x \sqrt{e^t+3}\, dt$

$g'(x) = -\frac{d}{dx}\int_7^x \sqrt{e^t+3}\, dt = -\sqrt{e^x+3}$ FTC Part 1

$g''(x) = \boxed{\frac{-1}{2\sqrt{e^x+3}} \cdot e^x}$ Regular Derivative Using Chain Rule

2. $f(x) = \sqrt{\cos(x^2+e^x)} + \cos\sqrt{x^2+e^x} + e^{\sqrt{x^2+\cos x}}$

$$f'(x) = \boxed{\frac{1}{2\sqrt{\cos(x^2+e^x)}}\left(-\sin(x^2+e^x)\right)(2x+e^x) - \sin\sqrt{x^2+e^x} \cdot \frac{1}{2\sqrt{x^2+e^x}}(2x+e^x) + e^{\sqrt{x^2+\cos x}} \cdot \frac{1}{2\sqrt{x^2+\cos x}} \cdot (2x-\sin x)}$$

3. $e^{xy} = 2 - xy$ Implicitly Differentiate

$\frac{d}{dx}\left(e^{xy}\right) = \frac{d}{dx}(2-xy)$

Chain + Product Rules Product Rule

$e^{xy}\left[x\frac{dy}{dx} + y\right] = -\left(x\frac{dy}{dx} + y(1)\right)$

$xe^{xy}\frac{dy}{dx} + ye^{xy} = -x\frac{dy}{dx} - y$

$(xe^{xy}+x)\frac{dy}{dx} = -y - ye^{xy}$

Solve $\frac{dy}{dx} = \boxed{\frac{-y-ye^{xy}}{xe^{xy}+x}}$

4. $f(x) = \dfrac{1+e^{-2x}}{1-e^{7x}}$ Quotient Rule

$$f'(x) = \frac{(1-e^{7x})(-2e^{-2x}) - (1+e^{-2x})(-7e^{7x})}{(1-e^{7x})^2}$$

$$= \frac{-2e^{-2x} + 2e^{5x} + 7e^{7x} + 7e^{5x}}{(1-e^{7x})^2}$$

$$= \boxed{\frac{-2e^{-2x} + 9e^{5x} + 7e^{7x}}{(1-e^{7x})^2}}$$

5. $$\int \frac{(1+e^{x})^2}{e^{x}}\,dx = \int \frac{1+2e^{x}+e^{2x}}{e^{x}}\,dx = \int \frac{1}{e^{x}} + \frac{2\cancel{e^{x}}}{\cancel{e^{x}}} + \frac{e^{2x}}{e^{x}}\,dx$$

Algebra Here
u-sub won't help.

$$= \int e^{-x} + 2 + e^{x}\,dx = \boxed{-e^{-x} + 2x + e^{x} + C}$$

6. $$\int (e^{x}+e^{-x})(e^{x}-e^{-x})\,dx$$

① Algebra First $= \int e^{2x} + 0 - e^{-2x}\,dx = \boxed{\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x} + C}$

OR

② u-sub. Next $= \int u\,du = \dfrac{u^2}{2} + C = \dfrac{(e^{x}+e^{-x})^2}{2} + C = \dfrac{1}{2}\left[e^{2x} + 2 + e^{-2x}\right] + C$

$\boxed{\begin{array}{l} u = e^{x}+e^{-x} \\ du = e^{x}-e^{-x}\,dx \end{array}}$

$$= \frac{1}{2}e^{2x} + 1 + \frac{1}{2}e^{-2x} + C$$

Absorb +1 into +C

They are equivalent

$= \boxed{\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x} + C^*}$ Match!

7. $\int \left(e^{4x} + e^{-9x}\right)^2 dx = \int \left(e^{4x}+e^{-9x}\right)\left(e^{4x}+e^{-9x}\right) dx$

Algebra

$= \int e^{8x} + 2e^{-5x} + e^{-18x}\, dx$

using K-rule $\int e^{kx}dx = \frac{1}{k}e^{kx}+C$

$= \boxed{\frac{1}{8}e^{8x} - \frac{2}{5}e^{-5x} - \frac{1}{18}e^{-18x} + C}$

8. $\int \frac{\sqrt{1+e^{-x}}}{e^x}\, dx = -\int \sqrt{u}\, du = \frac{-2}{3}u^{3/2} + C = \boxed{\frac{-2}{3}\left(1+e^{-x}\right)^{3/2} + C}$

$u = 1+e^{-x}$
$du = -e^{-x}dx$
$-du = \frac{1}{e^x}dx$

9. $\int \frac{we^{w^2}}{\left(17+e^{w^2}\right)^3}\, dw = \frac{1}{2}\int \frac{1}{u^3}\, du = \frac{1}{2}\left(\frac{u^{-2}}{-2}\right) + C = \frac{-1}{4u^2} + C = \boxed{\frac{-1}{4\left(17+e^{w^2}\right)^2} + C}$

u^{-3}

$u = 17+e^{w^2}$
$du = e^{w^2}(2w)\,dw$
$\frac{1}{2}du = we^{w^2}dw$

10. $\int \frac{e^{-1/x}}{7x^2}\, dx = \frac{1}{7}\int e^u\, du = \frac{1}{7}e^u + C = \boxed{\frac{1}{7}e^{-1/x} + C}$

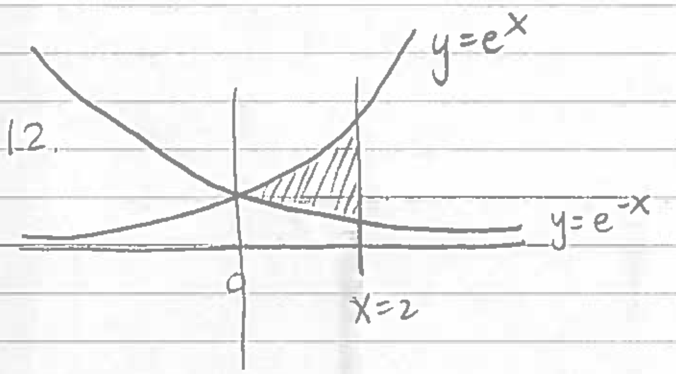
$u = -1/x = -x^{-1}$
$du = \frac{1}{x^2}dx$

11. $$\int \frac{e^{-2x}}{(1-e^{-2x})^{5/7}}\,dx = \frac{1}{2}\int \frac{1}{u^{5/7}}\,du = \frac{1}{2}\int u^{-5/7}\,du = \frac{1}{2}\,\frac{u^{2/7}}{\left(\frac{2}{7}\right)} + C$$

$$\boxed{\begin{aligned} u &= 1-e^{-2x} \\ du &= 2e^{-2x}dx \\ \tfrac{1}{2}\,du &= e^{-2x}dx \end{aligned}}$$

$$= \frac{1}{2}\left(\frac{7}{2}\right) u^{2/7} + C = \boxed{\frac{7}{4}\left(1-e^{-2x}\right)^{2/7} + C}$$

12.



$$\text{Area} = \int_0^2 e^x - e^{-x}\,dx = e^x + e^{-x}\Big|_0^2 = e^2 + e^{-2} - (\cancel{e^0}^{\,1} + \cancel{e^0}^{\,1})$$

$$= \boxed{e^2 + \frac{1}{e^2} - 2}$$

13. $$f(x) = \int \frac{e^{\sqrt{\tan x}}\sec^2 x}{\sqrt{\tan x}}\,dx = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{\tan x}} + C$$

$$\boxed{\begin{aligned} u &= \sqrt{\tan x} \\ du &= \frac{1}{2\sqrt{\tan x}}\cdot \sec^2 x\,dx \\ 2\,du &= \frac{\sec^2 x}{\sqrt{\tan x}}\,dx \end{aligned}}$$

Use condition $f(\pi/4) = 1$ to solve for C.

$$f(\pi/4) = 2e^{\cancel{\sqrt{\tan(\pi/4)}}^{\,1}} + C \overset{\text{set}}{=} 1$$

$$2e + C = 1 \Rightarrow C = 1-2e$$

Finally,

$$f(x) = \boxed{2e^{\sqrt{\tan x}} + (1-2e)}$$