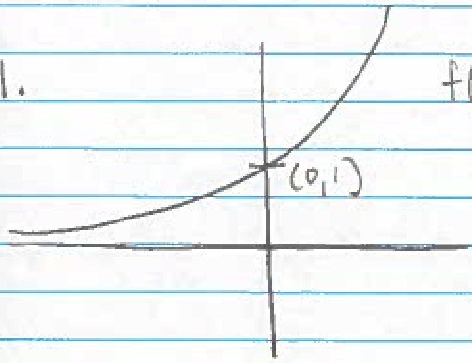
Worksheet 9 March 27^{th}, 2018

1. 

$f(x)=e^x$

Domain: $\mathbb{R}=(-\infty,\infty)$

Range: $(0,\infty)=\{y : y>0\}$

2. $\lim\limits_{x\to\infty} e^x=\boxed{\infty}$ $\qquad \lim\limits_{x\to-\infty} e^x=\boxed{0}$

3. (a) $f(x)=e^x \qquad f'(x)=\boxed{e^x}$

(b) $f(x)=\frac{1}{e^x}=e^{-x} \qquad f'(x)=e^{-x}(-1)=\boxed{\frac{-1}{e^x}}$

(c) $f(x)=e^{3x} \qquad f'(x)=e^{3x}(3)=\boxed{3e^{3x}}$

(d) $f(x)=\frac{1}{e^{7x}}=e^{-7x} \qquad f'(x)=e^{-7x}(-7)=\boxed{\frac{-7}{e^{7x}}}$

(e) $f(x)=e^{\sin x} \qquad f'(x)=\boxed{e^{\sin x}\cdot\cos x}$

(f) $f(x)=\sin(e^x) \qquad f'(x)=\boxed{\cos(e^x)\cdot e^x}$

(g) $f(x)=e^{\sqrt{x}} \qquad f'(x)=\boxed{e^{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}}$

(h) $f(x)=\sqrt{e^x}=(e^x)^{1/2} \qquad f'(x)=\frac{1}{2}(e^x)^{-1/2}\cdot e^x=\frac{e^x}{2\sqrt{e^x}}=\boxed{\frac{\sqrt{e^x}}{2}}$

OR

$f(x)=\sqrt{e^x}=(e^x)^{1/2}=e^{x/2} \qquad f'(x)=\boxed{e^{x/2}\cdot\frac{1}{2}}$

Match

(i) $f(x)=e^{(e^x)} \qquad f'(x)=\boxed{e^{(e^x)}\cdot e^x}$

(j) $f(x) = \underset{\text{Constant}}{e}$ $\quad f'(x) = \boxed{0}$

(k) $f(x) = \dfrac{e}{x} = ex^{-1}$ $\quad f'(x) = -ex^{-2} = \boxed{\dfrac{-e}{x^2}}$

(l) $f(x) = \dfrac{x}{e} = \dfrac{1}{e} \cdot x$ $\quad f'(x) = \boxed{\dfrac{1}{e}}$

(m) $f(x) = e^5$ $\quad f'(x) = \boxed{0}$ $\quad$ OR $\quad f'(x) = e^5 \cdot 0 = \boxed{0}$

constant

(n) $f(x) = ex$ $\quad f'(x) = \boxed{e}$

(o) $f(x) = \dfrac{1}{ex} = \dfrac{1}{e}x^{-1}$ $\quad f'(x) = -\dfrac{1}{e}x^{-2} = \boxed{\dfrac{-1}{ex^2}}$

(p) $f(x) = x^e$ $\quad f'(x) = \boxed{ex^{e-1}}$ $\quad$ Power Rule Here

Power Function

(q) $f(x) = \dfrac{1}{x^e} = x^{-e}$ $\quad f'(x) = \boxed{-ex^{-e-1}}$

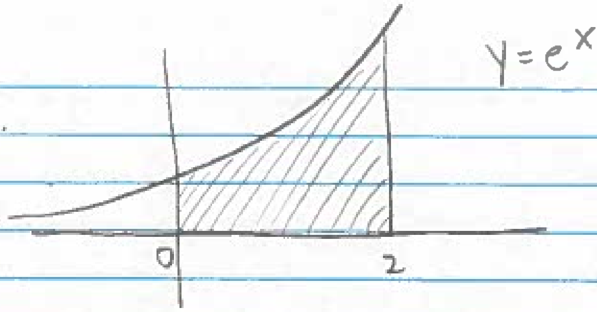
(r) $f(x) = \dfrac{e^{-2x}}{1+e^x}$ $\quad f'(x) = \dfrac{(1+e^x)e^{-2x}(-2) - e^{-2x}(e^x)}{(1+e^x)^2}$

Quotient Rule

$$= \frac{-2e^{-2x} - 2e^{-x} - e^{-x}}{(1+e^x)^2} = \boxed{\frac{-2e^{-2x} - 3e^{-x}}{(1+e^x)^2}}$$

(s) $f(x) = (e^{2x} - e^{-3x})^7$ $\quad f'(x) = \boxed{7(e^{2x} - e^{-3x})^6(2e^{2x} + 3e^{-3x})}$

4.



$$\text{Area} = \int_0^2 e^x\,dx = e^x\Big|_0^2 = e^2 - \cancel{e^0}^{\,1} = \boxed{e^2-1}$$

5. $e^{xy} = 2 + \sin x$ Implicit Differentiation

$$\frac{d}{dx}\left[e^{xy}\right] = \frac{d}{dx}\left[2+\sin x\right]$$

$$e^{xy}\left(x\cdot\frac{dy}{dx} + y\cdot(1)\right) = 0 + \cos x$$

OR Divide

$$xe^{xy}\frac{dy}{dx} + ye^{xy} = \cos x$$

$$xe^{xy}\frac{dy}{dx} = \cos x - ye^{xy} \Rightarrow \frac{dy}{dx} = \boxed{\frac{\cos x - ye^{xy}}{xe^{xy}}}$$

6. $$\int e^x\sqrt{1-e^x}\,dx = -\int \sqrt{u}\,du$$

$$\boxed{\begin{array}{l} u = 1-e^x \\ du = -e^x dx \\ -du = e^x dx \end{array}}$$

$$= -\int u^{1/2}\,du$$

$$= -\frac{u^{3/2}}{3/2} + C = \boxed{\frac{-2}{3}\left(1-e^x\right)^{3/2} + C}$$