WS #8 Math 106 Spring 2014

1. (a) $\int_1^5 7-x-x^2\,dx = 7x - \frac{x^2}{2} - \frac{x^3}{3}\Big|_1^5 = \left(35 - \frac{25}{2} - \frac{125}{3}\right) - \left(7 - \frac{1}{2} - \frac{1}{3}\right)$

$= 35 - \frac{25}{2} - \frac{125}{3} - 7 + \frac{1}{2} + \frac{1}{3}$

$= 28 - \frac{24}{2} - \frac{124}{3}$

$= 28 - 12 - \frac{124}{3}$

$= 16 - \frac{124}{3} = \frac{48}{3} - \frac{124}{3} = \boxed{\frac{-76}{3}}$

(b) Here $f(x) = 7-x-x^2$, $a=1$, $b=5$

$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$

$x_i = a + i\Delta x = 1 + \frac{4i}{n}$

$$\int_1^5 7-x-x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^n f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^n f\left(1+\frac{4i}{n}\right)\left(\frac{4}{n}\right)$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^n 7 - \left(1+\frac{4i}{n}\right) - \underbrace{\left(1+\frac{4i}{n}\right)^2}_{1+\frac{8i}{n}+\frac{16i^2}{n^2}} = \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^n 7 - 1 - \frac{4i}{n} - 1 - \frac{8i}{n} - \frac{16i^2}{n^2}$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^n 5 - \frac{12i}{n} - \frac{16i^2}{n^2} = \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^n 5 - \frac{4}{n} \sum_{i=1}^n \frac{12i}{n} - \frac{4}{n} \sum_{i=1}^n \frac{16i^2}{n^2}$$

$$= \lim_{n\to\infty} \frac{20}{n} \xrightarrow{n} \cancel{\sum_{i=1}^n 1} - \frac{48}{n^2} \sum_{i=1}^n i - \frac{64}{n^3} \sum_{i=1}^n i^2$$

$$= \lim_{n\to\infty} \frac{20}{\cancel{n}}(\cancel{n}) - \frac{48}{n^2}\left[\frac{n(n+1)}{2}\right] - \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} 20 - 24\left(\frac{\cancel{n}}{\cancel{n}}\right)\underbrace{\left(\frac{n+1}{n}\right)}_{1+1/n \to 1} - \frac{64}{6}\left(\frac{\cancel{n}}{\cancel{n}}\right)\underbrace{\left(\frac{n+1}{n}\right)}_{1+1/n \to 1}\underbrace{\left(\frac{2n+1}{n}\right)}_{2+1/n \to 2}$$

$$= 20 - 24 - \frac{64}{3} = -4 - \frac{64}{3} = \frac{-12}{3} - \frac{64}{3} = \boxed{\frac{-76}{3}}$$

2. $g(x) = \displaystyle\int_{3x}^{2} \frac{\cos t}{5+\cos t}\,dt$

$$g'(x) = \frac{d}{dx}\int_{3x}^{2} \frac{\cos t}{5+\cos t}\,dt = -\frac{d}{dx}\int_{2}^{3x} \frac{\cos t}{5+\cos t}\,dt = -\left[\frac{\cos(3x)}{5+\cos(3x)}\right](3) = \frac{-3\cos(3x)}{5+\cos(3x)}$$

$$g''(x) = \frac{[5+\cos(3x)](9\sin(3x)) - (-3\cos(3x))(-3\sin(3x))}{[5+\cos(3x)]^2}$$

$$= \frac{45\sin(3x) + \cancel{9\cos(3x)\sin(3x)} - \cancel{9\cos(3x)\sin(3x)}}{(5+\cos(3x))^2}$$

$$= \boxed{\frac{45\sin(3x)}{[5+\cos(3x)]^2}}$$

3. $$\int_{-1}^{2} |x-1| - 4\,dx = \int_{-1}^{1} \overset{-(x-1)}{\cancel{|x-1|}} - 4\,dx + \int_{1}^{2} \overset{x-1}{\cancel{|x-1|}} - 4\,dx$$

Recall $|x-1| = \begin{cases} x-1 & \text{if } x-1 \geq 0 \;\; (x \geq 1) \\ -(x-1) & \text{if } x-1 < 0 \;\; (x < 1) \end{cases}$

$$= \int_{-1}^{1} -x+1-4\,dx + \int_{1}^{2} x-1-4\,dx$$

$$= \int_{-1}^{1} -x-3\,dx + \int_{1}^{2} x-5\,dx$$

$$= \left.-\frac{x^2}{2} - 3x\right|_{-1}^{1} + \left.\frac{x^2}{2} - 5x\right|_{1}^{2}$$

$$= -\frac{1}{2} - 3 - \left(-\frac{1}{2} + 3\right) + 2 - 10 - \left(\frac{1}{2} - 5\right)$$

$$= \cancel{-\frac{1}{2}} - 3 + \cancel{\frac{1}{2}} - 3 - 8 - \frac{1}{2} + 5$$

$$= -6 - 3 - \frac{1}{2}$$

$$= -9 - \frac{1}{2} = \frac{-18}{2} - \frac{1}{2} = \boxed{\frac{-19}{2}}$$

4. $$\int_0^{\pi/6} \frac{\cos x}{(1+6\sin x)^2}\,dx = \frac{1}{6}\int_1^4 \frac{1}{u^2}\,du = \frac{1}{6}\left(\frac{u^{-1}}{-1}\right)\Bigg|_1^4 = \frac{-1}{6u}\Bigg|_1^4 = \frac{-1}{24} - \left(-\frac{1}{6}\right)$$

u^{-2}

$$= -\frac{1}{24} + \frac{4}{24}$$

$$= \frac{3}{24} = \boxed{\frac{1}{8}}$$

$$\boxed{\begin{aligned} u &= 1+6\sin x \\ du &= 6\cos x\,dx \\ \frac{1}{6}du &= \cos x\,dx \end{aligned}}$$

$$\boxed{\begin{aligned} x=0 &\Rightarrow u = 1+6\sin 0 = 1 \\ x=\pi/6 &\Rightarrow u = 1+6\sin\pi/6 \\ &= 1+6(1/2) \\ &= 1+3 \\ &= 4 \end{aligned}}$$

5. $$\int \frac{1}{u^2}\sqrt[3]{1-1/u}\,du = \int \sqrt[3]{w}\,dw = \int w^{1/3}\,dw = \frac{3}{4}w^{4/3} + C$$

$$= \boxed{\frac{3}{4}\left(1-1/u\right)^{4/3} + C}$$

$$\boxed{\begin{aligned} w &= 1-1/u \\ dw &= \frac{1}{u^2}du \end{aligned}}$$

6. $$\int x(3x-1)^{5/7}\,dx = \frac{1}{3}\int\left(\frac{u+1}{3}\right)u^{5/7}\,du = \frac{1}{9}\int (u+1)u^{5/7}\,du$$

$$\boxed{\begin{aligned} u &= 3x-1 \Rightarrow x = \frac{u+1}{3} \\ du &= 3\,dx \\ \frac{1}{3}du &= dx \end{aligned}}$$

$$= \frac{1}{9}\int u^{12/7} + u^{5/7}\,du = \frac{1}{9}\left[\frac{7}{19}u^{19/7} + \frac{7}{12}u^{12/7}\right] + C$$

$$= \boxed{\frac{1}{9}\left[\frac{7}{19}(3x-1)^{19/7} + \frac{7}{12}(3x-1)^{12/7}\right] + C}$$

7. $\displaystyle\int_{-\pi/2}^{\pi/2} \cos(3x)+\sin(5x)\,dx = \int_{-\pi/2}^{\pi/2} \cos(3x)\,dx + \int_{-\pi/2}^{\pi/2} \sin(5x)\,dx$

$u = 3x$
$du = 3\,dx$
$\frac{1}{3}du = dx$

$x = -\pi/2 \Rightarrow u = -\frac{3\pi}{2}$
$x = \frac{\pi}{2} \Rightarrow u = 3\pi/2$

$w = 5x$
$dw = 5\,dx$
$\frac{1}{5}dw = dx$

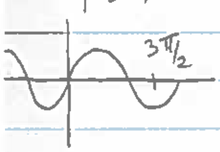
$x = -\pi/2 \Rightarrow w = -5\frac{\pi}{2}$
$x = \pi/2 \Rightarrow w = 5\pi/2$

$$= \frac{1}{3}\int_{-3\pi/2}^{\frac{3\pi}{2}} \cos u\,du + \frac{1}{5}\int_{-5\pi/2}^{\frac{5\pi}{2}} \sin w\,dw$$

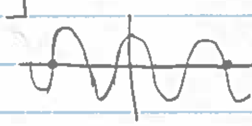
$$= \frac{1}{3}\sin u\,\Big|_{-3\pi/2}^{\frac{3\pi}{2}} - \frac{1}{5}\cos w\,\Big|_{-5\pi/2}^{5\pi/2}$$

$$= \frac{1}{3}\left[\overset{(-1)}{\cancel{\sin\left(\frac{3\pi}{2}\right)}} - \overset{1}{\cancel{\sin\left(\frac{-3\pi}{2}\right)}}\right] - \frac{1}{5}\left[\overset{0}{\cancel{\cos\left(\frac{5\pi}{2}\right)}} - \overset{0}{\cancel{\cos\left(\frac{-5\pi}{2}\right)}}\right]$$

y=sinx



y=cos x



$$= \frac{1}{3}\left[-1-1\right] \quad \cancel{-\frac{1}{5}\left[0\right]} \quad 0$$

Makes sense. 2^{nd} piece ODD.

$$= \boxed{\frac{-2}{3}}$$

8. $\displaystyle\int \frac{\sec^2\left(\frac{x}{3}\right)}{\tan^2\left(\frac{x}{3}\right)}\,dx = 3\int \frac{1}{u^2}\,du = 3\int u^{-2}\,du = \frac{3u^{-1}}{-1} + C$

$$= \frac{-3}{u} + C = \boxed{\frac{-3}{\tan\left(\frac{x}{3}\right)} + C}$$

$$\boxed{\begin{array}{l} u = \tan\left(\frac{x}{3}\right) \\ du = \frac{1}{3}\sec^2\left(\frac{x}{3}\right)dx \\ 3du = \sec^2\left(\frac{x}{3}\right)dx \end{array}}$$

9. Let $S(t)$ = amount of snow

(a) $\underbrace{S(2) - S(0)}_{\text{Net Change Snow}} = \displaystyle\int_0^2 S'(t)\,dt = \int_0^2 \frac{t}{(1+2t^2)^2}\,dt$

(b) $\displaystyle\int_0^2 \frac{t}{(1+2t^2)^2}\,dt = \frac{1}{4}\int_1^9 \frac{1}{u^2}\,du = -\frac{1}{4u}\Bigg|_1^9 = \frac{-1}{36} - \left(-\frac{1}{4}\right)$

$$\boxed{\begin{array}{l} u = 1+2t^2 \\ du = 4t\,dt \\ \frac{1}{4}du = t\,dt \end{array}} \quad \boxed{\begin{array}{l} t=0 \Rightarrow u=1 \\ t=2 \Rightarrow u=9 \end{array}} \quad = \frac{-1}{36} + \frac{9}{36} = \frac{8}{36} = \boxed{\frac{2}{9}} \text{ inch}$$

10. $v(t) = \sin t \qquad s(0) = 2$

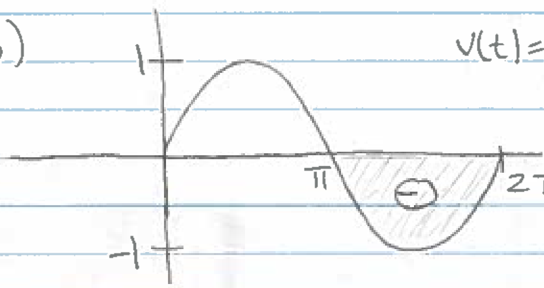
(a) $a(t) = v'(t) = \boxed{+\cos t}$

$$s(t) = \int v(t)\,dt = \int \sin t\,dt = -\cos t + C = \boxed{-\cos t + 3}$$

$$s(0) = -\overset{1}{\cancel{\cos 0}} + C \overset{\text{set}}{=} 2$$

$$-1 + C = 2 \Rightarrow C = 3$$

(b)

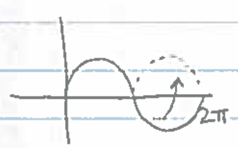


The object is not always moving to the right because between $t=\pi$ and $t=2\pi$ $v(t)$ is negative which means the position is decreasing then. That is moving in the negative direction or "to the left".

(c) Displacement $= \displaystyle\int_0^{2\pi} v(t)\,dt = \int_0^{2\pi} \sin t\,dt = -\cos t \Big|_0^{2\pi} = -\overset{1}{\cancel{\cos(2\pi)}} - (-\overset{1}{\cancel{\cos 0}})$

$$= -1 + 1 = \boxed{0}$$

$$\text{Total Distance} = \int_0^{2\pi} |v(t)|\,dt = \int_0^{2\pi} |\sin t|\,dt = \int_0^{\pi} \sin t\,dt + \int_{\pi}^{2\pi} -\sin t\,dt$$



$$= -\cos t \Big|_0^{\pi} + \cos t \Big|_{\pi}^{2\pi} = -\overset{(-1)}{\cancel{\cos \pi}} - (-\overset{1}{\cancel{\cos 0}}) + \overset{1}{\cancel{\cos 2\pi}} - \overset{(-1)}{\cancel{\cos \pi}}$$

$$= 1 + 1 + 1 + 1$$

$$= \boxed{4}$$