Math 106, Spring 2018

Worksheet 8, Tuesday, March 20th, 2018

Limit Definition of the Definite Integral

1. Compute $\displaystyle\int_1^5 7 - x - x^2 \, dx$ using two different methods:

   (a) Fundamental Theorem of Calculus

   (b) Limit Definition of the Definite Integral.

Differentiation

2. Compute $g''(x)$ where $g(x) = \displaystyle\int_x^2 \frac{\cos t}{5 + \cos t} \, dt$

Integration Evaluate each of the following integrals:

3. $\displaystyle\int_{-1}^{2} |x - 1| - 4 \, dx$

4. $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\cos x}{(1 + 6 \sin x)^2} \, dx$

5. $\displaystyle\int \frac{1}{u^2} \sqrt[3]{1 - \frac{1}{u}} \, du$ try using a w-substitution

6. $\displaystyle\int x(3x - 1)^{\frac{5}{7}} \, dx$

7. $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(3x) + \sin(5x) \, dx$ split this integral into 2 integrals with 2 u-substitutions

8. $\displaystyle\int \frac{\sec^2\left(\frac{x}{3}\right)}{\tan^2\left(\frac{x}{3}\right)} \, dx$

Displacement–Total Distance–Net Change

9. The intensity of a snow storm varies during the course of the storm. Assume that snow is falling at a rate of $\frac{t}{(1+2t^2)^2}$ inches per hour, where t is time in hours since the storm began.

   (a) Express total snowfall during the first two hours of the storm as a definite integral.

   (b) Compute the definite integral found in part (a).

10. Consider an object moving on the number line such that its velocity at time t is $\boldsymbol{v(t) = \sin t}$ feet per second. Also assume that $s(0) = 2$ feet, where as usual $s(t)$ is the position of the object at time t.

    (a) Compute the acceleration function $a(t)$ and the position function $s(t)$.

    (b) Draw the graph of $v(t)$ for $0 \le t \le 2\pi$, and explain why the object is *not* always moving to the right.

    (c) Compute the **displacement** and **total distance** travelled for $0 \le t \le 2\pi$.

Turn in your own solutions.