Math 106, Spring 2017

Worksheet 7, Tuesday, March 7th, 2017

Compute each of the following integrals:

1. $\displaystyle\int \sqrt{2x+7}\ dx = \frac{1}{2}\int \sqrt{u}\ du = \frac{1}{2}\left(\frac{2}{3}\right)u^{\frac{3}{2}} + C = \boxed{\frac{1}{3}(2x+7)^{\frac{3}{2}} + C}$

$$\boxed{\begin{aligned} u &= 2x+7 \\ du &= 2dx \\ \frac{1}{2}du &= dx \end{aligned}}$$

2. $\displaystyle\int \sec^2(3x)\ dx = \frac{1}{3}\int \sec^2 u\ du = \frac{1}{3}\tan u + C = \boxed{\frac{1}{3}\tan(3x) + C}$

$$\boxed{\begin{aligned} u &= 3x \\ du &= 3dx \\ \frac{1}{3}du &= dx \end{aligned}}$$

3. $\displaystyle\int \frac{1}{x^2\sqrt{9+\frac{1}{x}}}\ dx = -\int \frac{1}{\sqrt{u}}\ du = -\int u^{-\frac{1}{2}}\ du = -2u^{\frac{1}{2}} + C = \boxed{-2\left(9+\frac{1}{x}\right)^{\frac{1}{2}} + C}$

$$\boxed{\begin{aligned} u &= 9+\frac{1}{x} \\ du &= -\frac{1}{x^2}dx \\ -du &= \frac{1}{x^2}dx \end{aligned}}$$

4. $\displaystyle\int \frac{x}{(3x^2-8)^2}\ dx = \frac{1}{6}\int \frac{1}{u^2}\ du = \frac{1}{6}\int u^{-2}\ du = \frac{1}{6}\left(\frac{u^{-1}}{-1}\right) + C = -\frac{1}{6u} + C = \boxed{-\frac{1}{6(3x^2-8)} + C}$

$$\boxed{\begin{aligned} u &= 3x^2-8 \\ du &= 6xdx \\ \frac{1}{6}du &= xdx \end{aligned}}$$

5. $\displaystyle\int (\cos x)\ \sin^6 x\ dx = \int u^6\ du = \frac{u^7}{7} + C = \boxed{\frac{\sin^7 x}{7} + C}$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x dx \end{aligned}}$$

6. $\displaystyle\int \frac{\sin x}{\cos^5 x}\ dx = -\int \frac{1}{u^5}\ du = -\int u^{-5}\ du = -\frac{u^{-4}}{-4} + C = \frac{1}{4u^4} + C = \boxed{\frac{1}{4\cos^4 x} + C}$

$$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x dx \\ -du &= \sin x dx \end{aligned}}$$

7. $\displaystyle\int \sec^2 x \tan^3 x \; dx = \int u^3 \; du = \frac{u^4}{4} + C = \boxed{\frac{\tan^4 x}{4} + C}$

$$\boxed{\begin{aligned} u &= \tan x \\ du &= \sec^2 x dx \end{aligned}}$$

8. $\displaystyle\int \frac{\cos x}{(2+\sin x)^{\frac{5}{7}}} \; dx = \int \frac{1}{u^{\frac{5}{7}}} \; du = \int u^{-\frac{5}{7}} \; du = \frac{7}{2} u^{\frac{2}{7}} + C = \boxed{\frac{7}{2}(2+\sin x)^{\frac{2}{7}} + C}$

$$\boxed{\begin{aligned} u &= 2 + \sin x \\ du &= \cos x dx \end{aligned}}$$

9. $\displaystyle\int_1^4 \frac{1}{\sqrt{x}(1+\sqrt{x})^3} \; dx = 2\int_2^3 \frac{1}{u^3} \; du = 2\int_2^3 u^{-3} \; du = 2\left(\frac{u^{-2}}{-2}\right)\Bigg|_2^3$

$\displaystyle = -\frac{1}{u^2}\Bigg|_2^3 = -\frac{1}{3^2} - \left(-\frac{1}{2^2}\right) = -\frac{1}{9} + \frac{1}{4} = -\frac{4}{36} + \frac{9}{36} = \boxed{\frac{5}{36}}$

$$\boxed{\begin{aligned} u &= 1 + \sqrt{x} \\ du &= \frac{1}{2\sqrt{x}} dx \\ 2du &= \frac{1}{\sqrt{x}} dx \end{aligned}}$$

Change Limits: $\boxed{\begin{aligned} x = 1 &\Rightarrow u = 1 + \sqrt{1} = 1 + 1 = 2 \\ x = 4 &\Rightarrow u = 1 + \sqrt{4} = 1 + 2 = 3 \end{aligned}}$

10. $\displaystyle\int \frac{\tan\sqrt{x} \; \sec^2\sqrt{x}}{\sqrt{x}} \; dx = 2\int u \; du = u^2 + C = \boxed{(\tan\sqrt{x})^2 + C}$

$$\boxed{\begin{aligned} u &= \tan\sqrt{x} \\ du &= \sec^2\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right) dx \\ 2du &= \sec^2\sqrt{x}\left(\frac{1}{\sqrt{x}}\right) dx \end{aligned}}$$

11. Find a function $f(x)$ that satisfies $f'(x) = x^2 \sin(x^3)$ and $f(0) = 3$

$\displaystyle f(x) = \int x^2 \sin(x^3) \; dx = \frac{1}{3}\int \sin u \; du = -\frac{1}{3}\cos u + C = -\frac{1}{3}\cos(x^3) + C$

$$\boxed{\begin{aligned} u &= x^3 \\ du &= 3x^2 dx \\ \frac{1}{3}du &= x^2 dx \end{aligned}}$$

Next, use the point information $f(0) = 3$ to solve for C

$\displaystyle f(0) = -\frac{1}{3}\cos(0) + C \overset{\text{set}}{=} 3.$

So, $\displaystyle f(0) = -\frac{1}{3} + C = 3$ and we solve for $\displaystyle C = 3 + \frac{1}{3} = \frac{10}{3}.$

Finally, $f(x) = \boxed{-\frac{1}{3}\cos(x^3) + \frac{10}{3}}$.

12. Consider an object travelling with velocity given by $v(t) = t^2 - 3t + 2$ feet per second.

(a) Graph $v(t)$.
See me for a sketch. $v(t) = (t-2)(t-1)$ has zeroes at $t = 2$ and $t = 1$.

(b) Graph $|v(t)|$.
See me for a sketch.

(c) Write out the definition of $|v(t)|$.

$$|v(t)| = \begin{cases} t^2 - 3t + 2 & \text{if } t \leq 1 \text{ or } t \geq 2 \\ -(t^2 - 3t + 2) & \text{if } 1 \leq t \leq 2 \end{cases}$$

(d) Compute the **Displacement** for this object from time $t = 0$ to $t = 3$.
Displacement$= \int_0^3 v(t)\, dt = \int_0^3 t^2 - 3t + 2\, dt$
$= \frac{t^3}{3} - \frac{3t^2}{2} + 2t\Big|_0^3 = 9 - \frac{27}{2} + 6 - (0 - 0 + 0) = 15 - \frac{27}{2} = \frac{30}{2} - \frac{27}{2} = \boxed{\frac{3}{2}}$

(e) Compute the **Total Distance** for this object from time $t = 0$ to $t = 3$.
Total Distance$= \int_0^3 |v(t)|\, dt = \int_0^3 |t^2 - 3t + 2|\, dt$
$= \int_0^1 |t^2 - 3t + 2|\, dt + \int_1^2 |t^2 - 3t + 2|\, dt + \int_2^3 |t^2 - 3t + 2|\, dt$
$= \int_0^1 t^2 - 3t + 2\, dt + \int_1^2 -(t^2 - 3t + 2)\, dt + \int_2^3 t^2 - 3t + 2\, dt$
$= \frac{t^3}{3} - \frac{3t^2}{2} + 2t\Big|_0^1 + \left(-\frac{t^3}{3} + \frac{3t^2}{2} - 2t\right)\Big|_1^2 + \frac{t^3}{3} - \frac{3t^2}{2} + 2t\Big|_2^3$
$= \frac{1}{3} - \frac{3}{2} + 2 - (0 - 0 + 0) + \left(-\frac{8}{3} + 6 - 4 - (-\frac{1}{3} + \frac{3}{2} - 2)\right) + 9 - \frac{27}{2} + 6 - \left(\frac{8}{3} - 6 + 4\right)$
$= \frac{1}{3} - \frac{3}{2} + 2 - \frac{8}{3} + 2 + \frac{1}{3} - \frac{3}{2} + 2 + 9 - \frac{27}{2} + 6 - \frac{8}{3} + 2$
$= -\frac{14}{3} - \frac{33}{2} + 23 = -\frac{28}{6} - \frac{99}{6} + \frac{138}{6} = \boxed{\frac{11}{6}}$
OR you can use symmetry on the first and last pieces. Same area bounded between $t = 0$ and $t = 1$ as between $t = 2$ and $t = 3$, so just double one of the areas.

$$\int_0^3 |v(t)|\ dt = 2\int_0^1 |t^2 - 3t + 2|\ dt + \int_1^2 |t^2 - 3t + 2|\ dt = \dots$$