Math 106, Spring 2018

Answer Key

**Worksheet 5, Tuesday, February 20th, 2018**

2. Evaluate $\displaystyle\int_0^4 x - 1\ dx$ using Riemann Sums.

Here $f(x) = x - 1$, $a = 0$, $b = 4$, $\Delta x = \dfrac{4-0}{n} = \dfrac{b-a}{n} = \dfrac{4}{n}$
and $x_i = a + x_i = 0 + i\left(\dfrac{4}{n}\right) = \dfrac{4i}{n}$.

$$\begin{aligned}
\int_0^4 x - 1\ dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \quad &= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{4i}{n}\right)\frac{4}{n} \\
&= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{4i}{n} - 1\right)\frac{4}{n} \\
&= \lim_{n\to\infty}\left(\frac{4}{n}\sum_{i=1}^{n}\frac{4i}{n} - \frac{4}{n}\sum_{i=1}^{n} 1\right) \\
&= \lim_{n\to\infty}\left(\frac{16}{n^2}\sum_{i=1}^{n} i - \frac{4}{n}(n)\right) \\
&= \lim_{n\to\infty}\left(\frac{16}{n^2}\frac{n(n+1)}{2} - 4\right) \\
&= \lim_{n\to\infty}\left(8\left(\frac{n+1}{n}\right) - 4\right) \\
&= \lim_{n\to\infty}\left(8\left(\frac{n}{n} + \frac{1}{n}\right) - 4\right) \\
&= \lim_{n\to\infty}\left(8\left(1 + \frac{1}{n}\right) - 4\right) \\
&= 8 - 4 \\
&= \boxed{4}
\end{aligned}$$

3. Evaluate $\int_0^2 x^2 - 5x\ dx$ using Riemann Sums.

Here $f(x) = x^2 - 5x,\ a = 0,\ b = 2,\ \Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$ and $x_i = a + x_i = 0 + i\left(\frac{2}{n}\right) = \frac{2i}{n}$.

$$\begin{aligned}
\int_0^2 x^2 - 5x\ dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \quad &= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\frac{2}{n} \\
&= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\left(\frac{2i}{n}\right)^2 - 5\left(\frac{2i}{n}\right)\right)\frac{2}{n} \\
&= \lim_{n\to\infty}\left(\frac{2}{n}\sum_{i=1}^{n}\frac{4i^2}{n^2} - \frac{2}{n}\sum_{i=1}^{n}\frac{10i}{n}\right) \\
&= \lim_{n\to\infty}\left(\frac{8}{n^3}\sum_{i=1}^{n} i^2 - \frac{20}{n^2}\sum_{i=1}^{n} i\right) \\
&= \lim_{n\to\infty}\left(\frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{20}{n^2}\frac{n(n+1)}{2}\right) \\
&= \lim_{n\to\infty}\left(\frac{8}{6}\left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) - \frac{20}{2}\left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right)\right) \\
&= \lim_{n\to\infty}\left(\frac{4}{3}(1)\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - 10(1)\left(1+\frac{1}{n}\right)\right) \\
&= \frac{8}{3} - 10 = \frac{8}{3} - \frac{30}{3} = \boxed{-\frac{22}{3}}
\end{aligned}$$