Worksheet #4 $\quad 9x^{-8}$

1. (a) $f(x) = \sin\left(\cos^6\left(9/x^8\right)\right) = \sin\left[\left(\cos\left(9/x^8\right)\right)^6\right]$

$$\boxed{f'(x) = \cos\left(\cos^6\left(9/x^8\right)\right)\cdot 6\cos^5\left(9/x^8\right)\left(-\sin\left(9/x^8\right)\right)\cdot\left(-72x^{-9}\right)}$$

(b) $y = \tan\left(\frac{9}{\sin x}\right) = \tan\left[9(\sin x)^{-1}\right]$

$$\boxed{y' = \sec^2\left(\frac{9}{\sin x}\right)\left(-9(\sin x)^{-2}\right)\cos x}$$

(c) $g(t) = \dfrac{\sec(3t) + \sec^2(7t)}{\cos t + 1}$

$$\boxed{g'(t) = \frac{(\cos t+1)\left[\sec(3t)\tan(3t)3 + 2\sec(7t)\sec(7t)\tan(7t)7\right] - \left[\sec(3t)+\sec^2(7t)\right]\left[-\sin t\right]}{(\cos t+1)^2}}$$

2. $f(x) = \tan x \sin x - \dfrac{\sin(2x)}{2}$ $\qquad 2\cdot\pi/6 = \pi/3$

$f'(x) = \tan x\cos x + \sin x\cdot\sec^2 x - \frac{1}{\cancel{2}}\cos(2x)(\cancel{2})$

$f'(\pi/6) = \tan(\pi/6)\cos(\pi/6) + \sin(\pi/6)\sec^2(\pi/6) - \cos(\pi/3)$

$\cos\pi/6 = \sqrt{3}/2$

$\sec\pi/6 = \dfrac{2}{\sqrt{3}}$

$= \cancel{\dfrac{1}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{2}} + \dfrac{1}{2}\left(\dfrac{2}{\sqrt{3}}\right)^2 - \cancel{\dfrac{1}{2}}$

cancel

$= \dfrac{1}{2}\left(\dfrac{4}{3}\right) = \boxed{\dfrac{2}{3}}$

3. $\sin(x+y) = 2x - 2y$ at point (π, π)

$$\cos(x+y)\left[1+\frac{dy}{dx}\right] = 2 - 2\frac{dy}{dx}$$

$$\cos(\pi+\pi)\left[1+\frac{dy}{dx}\right] = 2 - 2\frac{dy}{dx}$$

$$\cos 2\pi$$

$$1$$

$$1 + \frac{dy}{dx} = 2 - 2\frac{dy}{dx}$$

$$3\frac{dy}{dx} = 1 \Rightarrow \left.\frac{dy}{dx}\right|_{(\pi,\pi)} = \frac{1}{3}$$

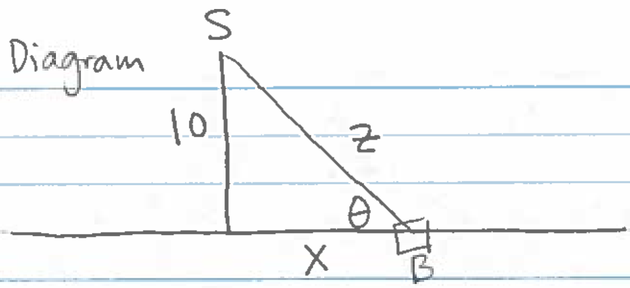
Equation of Tangent Line

$$y - \pi = \tfrac{1}{3}(x - \pi) \qquad y = \tfrac{1}{3}x - \tfrac{\pi}{3} + \pi$$

$$\frac{3\pi}{3}$$

$$\boxed{y = \frac{1}{3}x + \frac{2\pi}{3}}$$

4. • Diagram



• Variables

Let x = distance from point on track closest to Sally at time t
z = distance between Sally and Bob at time t.
θ = angle Bob's head rotated from track line

Find $\frac{dx}{dt} = ?$ when $\frac{d\theta}{dt} = -2$ rad./sec. and $z = 13$

shrinking

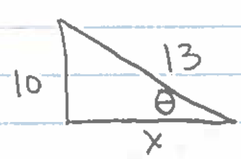
• Equation

$$\tan\theta = \frac{10}{x}$$

• Differentiate.

$$\sec^2\theta \frac{d\theta}{dt} = \frac{-10}{x^2}\frac{dx}{dt}$$

• Extra Solvable Information



$x = \sqrt{169-100} = \sqrt{69} \Rightarrow \sec\theta = \frac{13}{\sqrt{69}}$

• Substitute

$$\left(\frac{13}{\sqrt{69}}\right)^2(-2) = \frac{-10}{(\sqrt{69})^2}\frac{dx}{dt}$$

• Solve

$$\frac{dx}{dt} = \frac{169}{\cancel{69}}\cancel{(-2)}\left(\frac{-\cancel{69}}{\cancel{100}_{50}}\right) = \frac{169}{50}$$

• Answer The train is travelling at $169/50$ meters per second at that moment

5. $f''(x) = 20x^3 + 12x^2 + 4$

$f'(x) = 5x^4 + 4x^3 + 4x + C_1$

$f(x) = x^5 + x^4 + 2x^2 + C_1 x + C_2$

First use $f(0) = 8$

$f(0) = 0 + 0 + 0 + 0 + C_2 \overset{\text{set}}{=} 8 \Rightarrow C_2 = 8$

$\Rightarrow f(x) = x^5 + x^4 + 2x^2 + C_1 x + 8$

Next use $f(1) = 5$

$f(1) = 1 + 1 + 2 + C_1 + 8 \overset{\text{set}}{=} 5 \Rightarrow C_1 = -7$

Finally, $\boxed{f(x) = x^5 + x^4 + 2x^2 - 7x + 8}$

6. (a) $$\frac{7x^{2/5} + 8x^{-4/3} + \frac{1}{x}}{\sqrt{x}} = \frac{7x^{2/5}}{x^{1/2}} + \frac{8x^{-4/3}}{x^{1/2}} + \frac{x^{-1}}{x^{1/2}} = 7x^{-1/10} + 8x^{-11/6} + x^{-3/2}$$

$\frac{2}{5} - \frac{1}{2}$ $\quad -\frac{4}{3} - \frac{1}{2}$ $\quad -1 - \frac{1}{2}$

$\frac{4}{10} - \frac{5}{10}$ $\quad -\frac{8}{6} - \frac{3}{6}$

antidifferentiate.

$$\frac{7x^{9/10}}{\left(\frac{9}{10}\right)} + \frac{8x^{-5/6}}{\left(-\frac{5}{6}\right)} + \frac{x^{-1/2}}{\left(-\frac{1}{2}\right)} + C$$

$$= \boxed{\frac{70}{9}x^{9/10} - \frac{48}{5}x^{-5/6} - 2x^{-1/2} + C}$$

$3 + \frac{2}{7}$

$\frac{23}{7}$

6.

(b) $\left(\sqrt{3} + \frac{1}{x^3}\right)\left(x - \frac{1}{x^{2/7}}\right) = \sqrt{3}\,x - \sqrt{3}\,x^{-2/7} + x^{-2} - x^{-23/7}$

antidifferentiate.

$$\frac{\sqrt{3}}{2}x^2 - \frac{\sqrt{3}\,x^{5/7}}{(5/7)} + \frac{x^{-1}}{(-1)} - \frac{x^{-16/7}}{\left(-16/7\right)} + C$$

$$\boxed{\frac{\sqrt{3}}{2}x^2 - \frac{7\sqrt{3}}{5}x^{5/7} - \frac{1}{x} + \frac{7}{16}x^{-16/7} + C}$$

7. 

$v(0) = {}^{+}80$ ft/sec (up!) $\qquad s_0 = ?$

$v(\text{impact}) = -112$ ft/sec. (down!)

Equations for Falling Bodies

$a(t) = -32$

$v(t) = -32t + v_0 = -32t + 80$

$s(t) = -16t^2 + 80t + \boxed{s_0} \longleftarrow$ looking for.

$32\overline{)192}$ → 6

$v(\text{impact}) = -32t + 80 \overset{\text{set}}{=} -112$

$-32t = -192 \Rightarrow t_{\text{impact}} = 6$ seconds

$\Rightarrow s(6) = 0$

$s(6) = -16(6)^2 + 80(6) + s_0 \overset{\text{set}}{=} 0.$

$\underbrace{-576 + 480}_{-96} + s_0 = 0 \Rightarrow s_0 = 96$

16
36
96
80
76

Answer: The Bridge was 96 feet high.