Math 106, Spring 2018

Worksheet 3, Tuesday, February 6th, 2018

Definition: Take F, f functions defined on an interval I and suppose that $F'(x) = f(x)$ on I. Then
- $F(x)$ is called **an** antiderivative of $f(x)$
- $F(x)+C$ is called the **most general** antiderivative of $f(x)$, where C is any constant Real Number.

We will use the notation $\int f(x)\ dx$ to denote the most general antiderivative.

For example: $\int x^7\ dx = \frac{x^8}{8} + C$ where $+C$ represent all possible constants.

Note that $\frac{x^8}{8} + 3$ is **an** antiderivative of x^7. So is $\frac{x^8}{8} + 2014$ as well as $\frac{x^8}{8} - 5$ and $\frac{x^8}{8} + \sqrt{3}$.

Hint: if you ever want to know whether you found the correct antiderivative, take the derivative of your answer and check that you return to the original function.

1. Write a *general power rule* for $\int x^n\ dx$ where n is any real number with $n \neq -1$.

   (We will learn the $n = -1$ case at the very end of this semester.)

2. Compute $\int \sqrt{x}\ dx$
3. Compute $\int \frac{1}{x^9}\ dx$
4. Compute $\int \frac{1}{\sqrt{x}}\ dx$
5. Compute $\int \frac{1}{x^{\frac{3}{7}}}\ dx$
6. Compute $\int \cos x\ dx$
7. Compute $\int \sin x\ dx$
8. Compute $\int \sec^2 x\ dx$
9. Compute $\int \sec x \tan x\ dx$

10. Find the general antiderivative of the following functions:

   (a) $f(x) = x^3 + \dfrac{1}{\sqrt{x}} + 2$

   (b) $f(x) = x^3(1 + x^2)$

   (c) $f(x) = \dfrac{x + \sqrt{x} + 7}{x^3}$

   (d) $f(x) = x^2 + x(1 + x)^2$

   (e) $f(w) = \dfrac{w + \sqrt{w}}{\sqrt[4]{w^3}}$

   (f) $f(x) = -3\cos x - \sec^2 x - 7\sec x \tan x$

11. Is $\frac{1}{6}(x^2 + 1)^3$ an antiderivative of $x(x^2 + 1)^2$? Hint: use the definition of antiderivative...

12. Consider the curve $y = 2x + \sin x$. Explain why the tangent lines of this curve are never horizontal.

13. Find an antiderivative $F(x)$ of $2 + \sin x$ that satisfies $F\left(\frac{\pi}{2}\right) = 3$.

14. Find the indicated functions:

   (a) $f(x)$ where $f'(x) = x^2 + 1$ and $f(1) = 3$.

   (b) $g(t)$ where $g'(t) = t(2 + \sqrt{t})$ and $g(4) = 30$.

   (c) $h(t)$ where $h''(t) = \dfrac{1}{\sqrt{t}} + 3t^2$ and $h'(1) = 2$, $h(1) = 0$.

15. CHALLENGE: Can you use a *guess and check* approach to compute the function $f(x)$ where $f'(x) = \dfrac{\sec x \tan x}{\sqrt{\sec x + 8}}$ and $f(0) = 7$? Check your answer.

Turn in your own solutions.