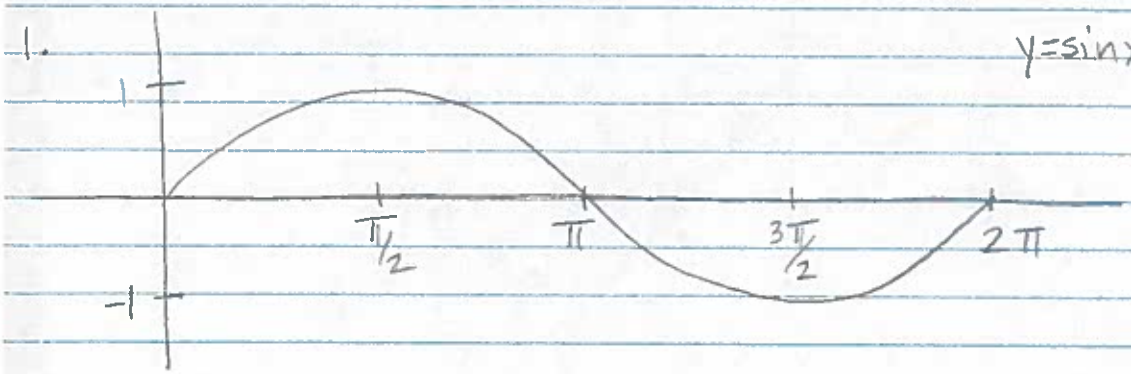
Worksheet #1 Spring 2017

1.



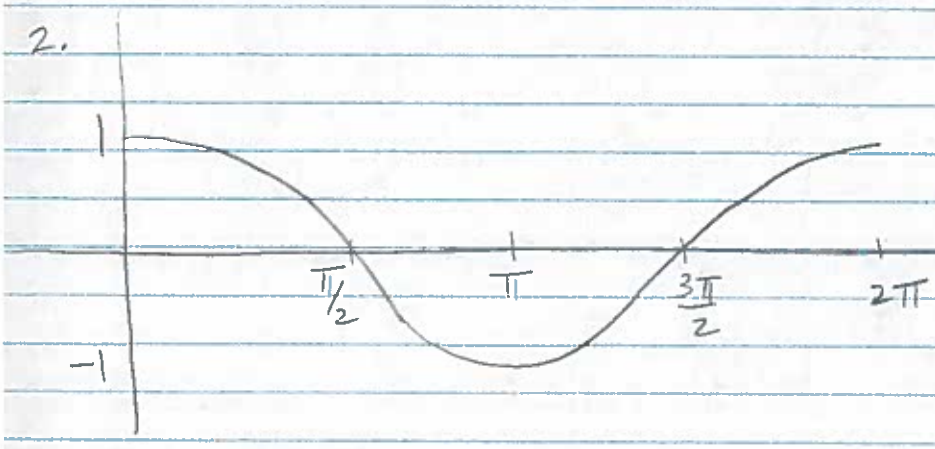
a. $\sin 0 = 0$

b. $\sin \pi = 0$

c. $\sin \frac{\pi}{2} = 1$

d. $\sin \frac{3\pi}{2} = -1$

2.



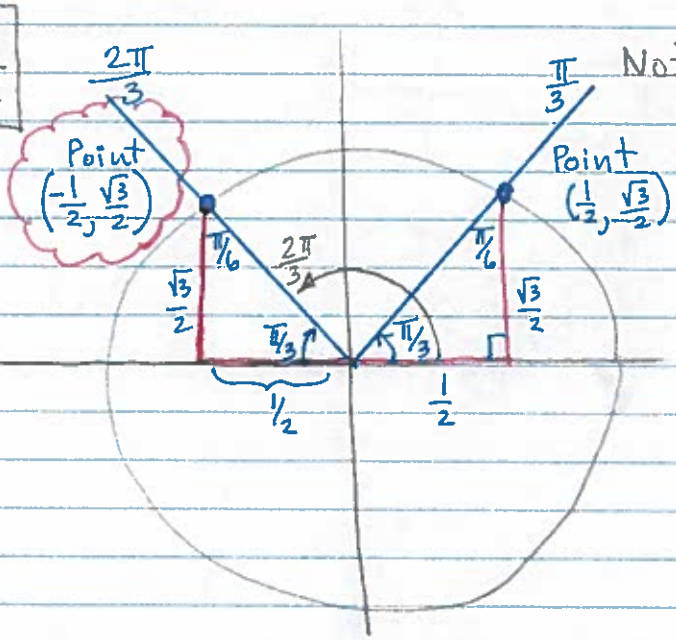
a. $\cos 0 = 1$

b. $\cos \pi = -1$

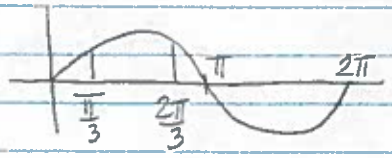
c. $\cos \frac{\pi}{2} = 0$

d. $\cos \frac{3\pi}{2} = 0$

3a. $\sin \frac{2\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$



Note: $\sin \frac{2\pi}{3} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$



b. $\cos \frac{2\pi}{3} = \boxed{\frac{-1}{2}}$

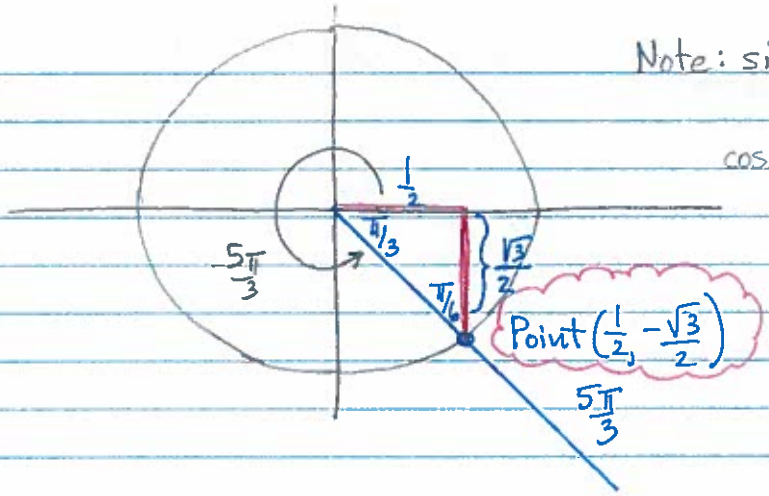
$\cos \frac{2\pi}{3} = -\cos \frac{\pi}{3} = -\frac{1}{2}$

3c. $\sin\frac{5\pi}{3} = \boxed{\frac{-\sqrt{3}}{2}}$

Note: $\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

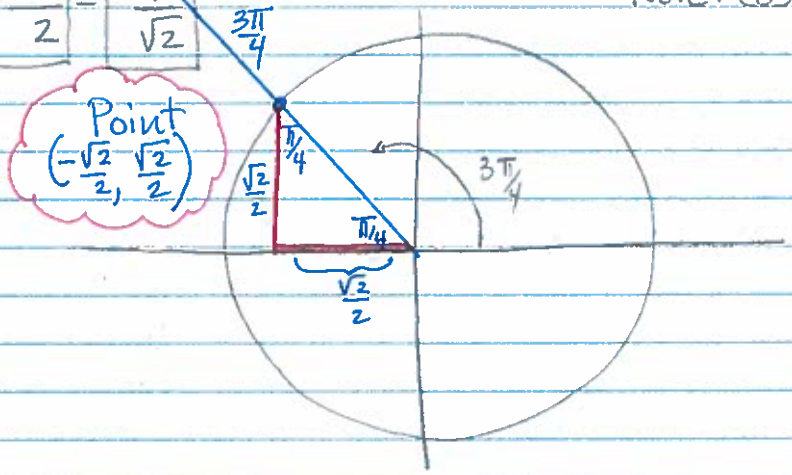
$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$

d. $\cos\frac{5\pi}{3} = \boxed{\frac{1}{2}}$

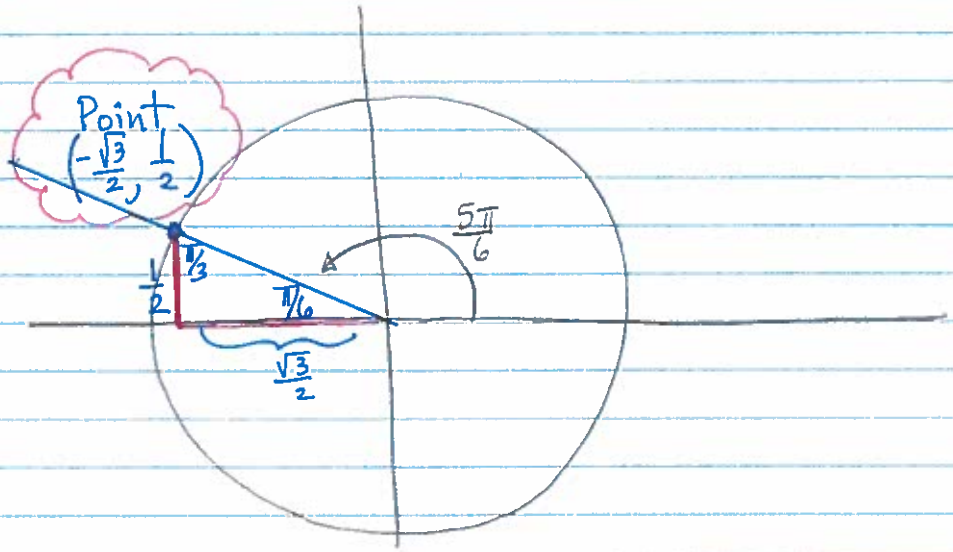


e. $\cos\frac{3\pi}{4} = \boxed{\frac{-\sqrt{2}}{2}} = \boxed{\frac{-1}{\sqrt{2}}}$

Note: $\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

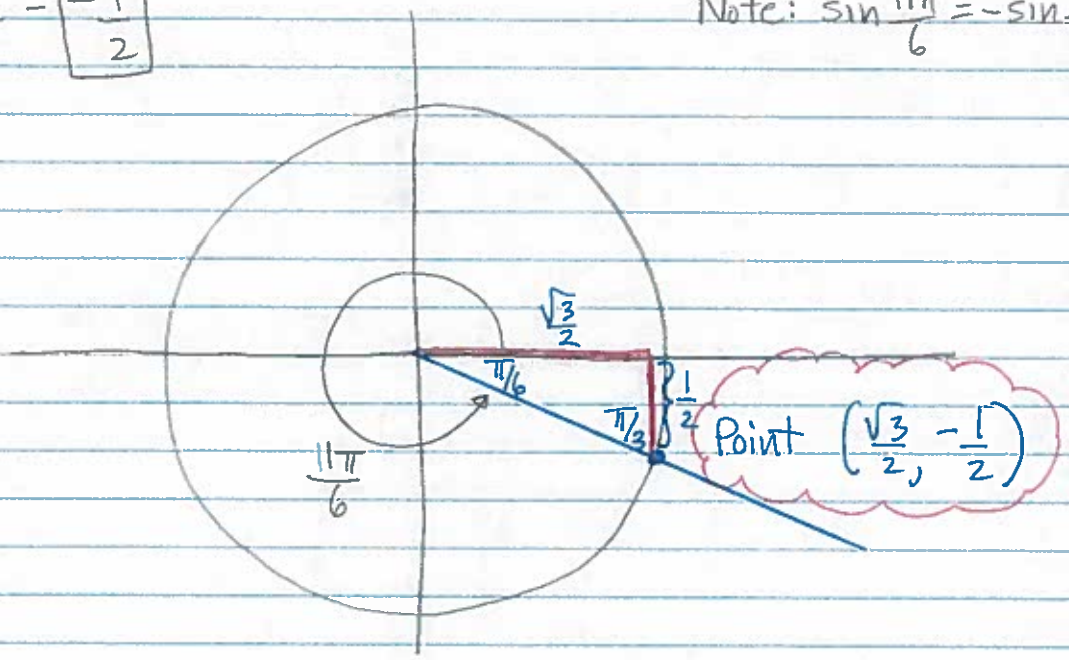


f. $\tan\frac{5\pi}{6} = \frac{\sin\frac{5\pi}{6}}{\cos\frac{5\pi}{6}} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} \quad \frac{-2}{\sqrt{3}} = \boxed{\frac{-1}{\sqrt{3}}}$



3g. $\sin\frac{11\pi}{6} = \boxed{-\frac{1}{2}}$

Note: $\sin\frac{11\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$



4a. $f(x) = \sin x$

$f'(x) = \cos x$

Point: $(0, f(0)) = (0, \sin 0) = (0, 0)$

Specific Slope: $f'(0) = \cos 0 = 1$

Point Slope Form: $y - 0 = f'(0)(x - 0)$

$\boxed{y = x}$ ← Equation of Tangent Line

4b. $f(x) = \cos x$

$f'(x) = -\sin x$

Point: $\left(\frac{\pi}{6}, f\left(\frac{\pi}{6}\right)\right) = \left(\frac{\pi}{6}, \cos\frac{\pi}{6}\right) = \left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$

Specific Slope: $f'\left(\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

Point Slope Form: $y - \frac{\sqrt{3}}{2} = -\frac{1}{2}\left(x - \frac{\pi}{6}\right)$

$$\boxed{y = -\frac{1}{2}x + \frac{\pi}{12} + \frac{\sqrt{3}}{2}}$$

4c. $f(x) = \tan x$

$f'(x) = \sec^2 x$

Point: $\left(\frac{\pi}{3}, f\left(\frac{\pi}{3}\right)\right) = \left(\frac{\pi}{3}, \tan\frac{\pi}{3}\right) = \left(\frac{\pi}{3}, \sqrt{3}\right)$

Specific Slope: $f'\left(\frac{\pi}{3}\right) = \sec^2\left(\frac{\pi}{3}\right) = \frac{1}{\cos^2\left(\frac{\pi}{3}\right)} = \frac{1}{\left[\cos\frac{\pi}{3}\right]^2} = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4$

Point Slope Form: $y - \sqrt{3} = 4\left(x - \frac{\pi}{3}\right)$

$$\boxed{y = 4x - \frac{4\pi}{3} + \sqrt{3}}$$

5. Show $\frac{d}{dx}\tan x = \sec^2 x$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot \frac{d}{dx}\sin x - \sin x \cdot \frac{d}{dx}\cos x}{\cos^2 x} \quad \text{Quotient Rule}$$

$$= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \to 1 \text{ ← Identity } \cos^2 x + \sin^2 x = 1$$

$$= \frac{1}{\cos^2 x}$$

$$= \frac{1}{(\cos x)^2}$$

$$= \left(\frac{1}{\cos x}\right)^2$$

$$= \sec^2 x \quad \checkmark$$

6. Show $\frac{d}{dx}\sec x = \sec x \tan x$

Quotient Rule

$$\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x} = \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x}$$

$$= \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} = \sec x \cdot \tan x \quad \checkmark$$

OR

chain rule

$$\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x} = \frac{d}{dx}(\cos x)^{-1} = (-1)(\cos x)^{-2}(-\sin x)$$

$$= \frac{+1}{\cos^2 x}\cdot\sin x = \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} = \sec x \cdot \tan x \quad \checkmark$$

7. $W(x) = \cos^2(2x) + \tan(2x) + 3\sec x$

$$W'(x) = 2\cos(2x)(-\sin(2x))\cdot 2 + \sec^2(2x)\cdot 2 + 3\sec x\tan x$$

$$= -4\cos(2x)\sin(2x) + 2\sec^2(2x) + 3\sec x\tan x$$

$$W'(\pi/6) = -4\cos\left[2(\pi/6)\right]\cdot\sin\left[2(\pi/6)\right] + 2\sec^2\left[2(\pi/6)\right] + 3\sec(\pi/6)\cdot\tan(\pi/6)$$

$$= -4\cos(\pi/3)\sin(\pi/3) + 2\sec^2(\pi/3) + 3\sec(\pi/6)\tan(\pi/6)$$

see below; see below

$$= -4\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + 2\cdot(2)^2 + 3\left(\frac{2}{\sqrt{3}}\right)\cdot\frac{1}{\sqrt{3}}$$

$$= -\sqrt{3} + 8 + 2 = \boxed{10 - \sqrt{3}}$$

Note: $\sec(\pi/3) = \frac{1}{\cos(\pi/3)} = \frac{1}{1/2} = 2$

$\sec(\pi/6) = \frac{1}{\cos(\pi/6)} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$ ($\cos(\pi/6) \to \sqrt{3}/2$)