Name: Answer Key

Math 106 Quiz #9 April 9, 2018

- This is a closed-book quiz. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

1. [10 points] Find the function $f(x)$ that satsifies $f'(x) = \dfrac{e^x + \cos x}{\sqrt{e^x + \sin x}}$ and $f(0) = 5$.

$$f(x) = \int \frac{e^x + \cos x}{\sqrt{e^x + \sin x}}\,dx = \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{u} + C$$

$$= 2\sqrt{e^x + \sin x} + C$$

$$\boxed{\begin{array}{l} u = e^x + \sin x \\ du = e^x + \cos x\,dx \end{array}}$$

Use $f(0) = 2\sqrt{\cancel{e^0}^{\,1} + \sin 0} + C \overset{\text{set}}{=} 5$

$2 + C = 5 \Rightarrow C = 3$

Finally, $f(x) = \boxed{2\sqrt{e^x + \sin x} + 3}$

2. [10 points] Compute $\displaystyle\int_4^9 \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\,dx = 2\int_2^3 \frac{1}{e^u}\,du = 2\int_2^3 e^{-u}\,du$

$$\boxed{\begin{array}{l} u = \sqrt{x} \\ du = \dfrac{1}{2\sqrt{x}}\,dx \\ 2\,du = \dfrac{1}{\sqrt{x}}\,dx \end{array}} \quad \boxed{\begin{array}{l} x = 4 \Rightarrow u = \sqrt{4} = 2 \\ x = 9 \Rightarrow u = \sqrt{9} = 3 \end{array}}$$

$$= -2e^{-u}\Big|_2^3$$

$$= -2\left[e^{-3} - e^{-2}\right]$$

$$= \boxed{\frac{2}{e^2} - \frac{2}{e^3}}$$

3. [10 points] Compute $\int \frac{(2)}{e^{2x}(e^{-2x}+2)^2}\,dx = -\frac{\cancel{2}}{\cancel{2}}\int \frac{1}{u^2}\,du = -\int u^{-2}\,du$

$$\boxed{\begin{aligned} u &= e^{-2x}+2 \\ du &= -2e^{-2x}dx \\ -\frac{1}{2}\,du &= \frac{1}{e^{2x}}\,dx \end{aligned}}$$

$$= \cancel{-}\frac{u^{-1}}{\cancel{-1}} + C = \frac{1}{u} + C = \boxed{\frac{1}{e^{-2x}+2} + C}$$

4. [10 points] Compute $\int \left(e^{3x} + \frac{1}{e^{8x}}\right)^2 dx$

$$= \int (e^{3x} + e^{-8x})(e^{3x} + e^{-8x})\,dx$$

$$= \int e^{6x} + 2e^{-5x} + e^{-16x}\,dx$$

$$= \frac{e^{6x}}{6} + \frac{2e^{-5x}}{-5} + \frac{e^{-16x}}{-16} + C$$

$$= \boxed{\frac{e^{6x}}{6} - \frac{2}{5e^{5x}} - \frac{1}{16e^{16x}} + C}$$

BONUS [2 points] Compute the derivative for $y = \dfrac{1}{\tan(e^x + \sqrt{x})} + \dfrac{1}{e^{\sec\sqrt{x}}} + \dfrac{1}{\sqrt{e^{\cos x} + 9}}$.

$$y = \left(\tan\left(e^x + \sqrt{x}\right)\right)^{-1} + e^{-\sec\sqrt{x}} + \left(e^{\cos x} + 9\right)^{-1/2}$$

$$y' = -\left[\tan(e^x + \sqrt{x})\right]^{-2} \cdot \sec^2(e^x + \sqrt{x}) \cdot \left(e^x + \frac{1}{2\sqrt{x}}\right)$$

$$+ e^{-\sec\sqrt{x}}\left[-\sec\sqrt{x}\tan\sqrt{x}\right] \cdot \frac{1}{2\sqrt{x}}$$

$$-\frac{1}{2}\left[e^{\cos x} + 9\right]^{-3/2}\left(e^{\cos x} \cdot (-\sin x) + 0\right)$$