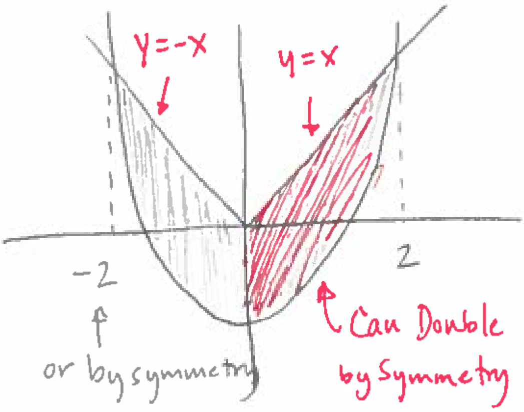
Name: Answer Key

Math 106 Quiz #8 April 2, 2018

- This is a closed-book quiz. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

1. [10 points] Find the area enclosed by $y = |x|$ and $y = x^2 - 2$. Sketch the region.



$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Intersect?

$x = x^2 - 2$

Solve $x^2 - x - 2 = 0$ $(x \geq 0)$ and $x < 0$

$(x-2)(x+1) = 0$

$x = 2$ $\cancel{x = -1}$

$-x = x^2 - 2$

$x^2 + x - 2$

$(x+2)(x-1) = 0$

$x = -2$ $\cancel{x = 1}$

Cases:

$$\text{Area} = \int_{-2}^{0} \overset{-x}{\cancel{|x|}} - (x^2-2)\,dx + \int_0^2 \overset{x}{\cancel{|x|}} - (x^2-2)\,dx$$

$$\underline{\underline{\text{OR}}} = 2\int_0^2 x - (x^2-2)\,dx = 2\int_0^2 x - x^2 + 2\,dx = 2\left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]\Bigg|_0^2$$

Double using Symmetry

$$= 2\left[\left(\frac{\cancel{4}^2}{\cancel{2}} - \frac{8}{3} + 4\right) - (0 - 0 + 0)\right]$$

$$= 2\left[6 - 8/3\right] = 2\left[\frac{18}{3} - \frac{8}{3}\right] = 2\left[\frac{10}{3}\right] = \boxed{\frac{20}{3}}$$

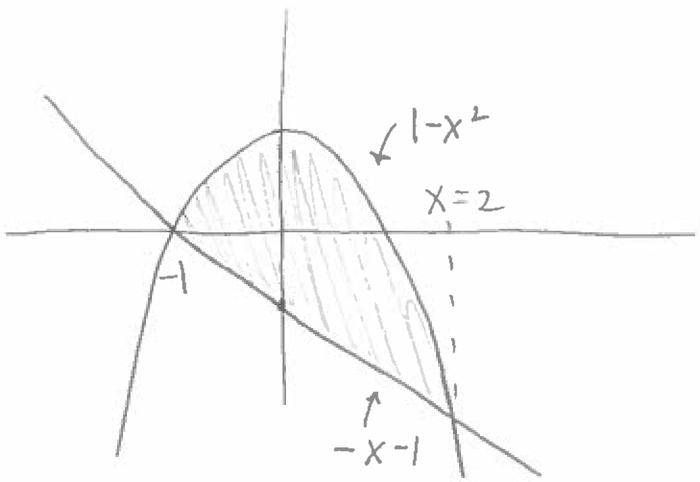
$\underline{\underline{\text{OR}}}$ Work BOTH

$$\int_{-2}^{0} -x - x^2 + 2\,dx + \int_0^2 x - x^2 + 2\,dx = -\frac{x^2}{2} - \frac{x^3}{3} + 2x\Bigg|_{-2}^{0} + \frac{x^2}{2} - \frac{x^3}{3} + 2x\Bigg|_0^2$$

$$= (0 - 0 + 0) - \left(2 + \frac{8}{3} - 4\right) + (2 - 8/3 + 4) - (0 - 0 + 0)$$

$$= 6 - 8/3 + 6 - 8/3 = 12 - \frac{16}{3} = \frac{36}{3} - \frac{16}{3} = \boxed{\frac{20}{3}}$$

2. [10 points] Find the area enclosed by $y = -x - 1$ and $y = 1 - x^2$. Sketch the region.



Intersect ?

$$-x-1 = 1-x^2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \quad x = -1$$

$$\text{Area} = \int_{-1}^{2} 1 - x^2 - (-x-1)\,dx = \int_{-1}^{2} 1 - x^2 + x + 1\,dx = \int_{-1}^{2} -x^2 + x + 2\,dx$$

$$= \frac{-x^3}{3} + \frac{x^2}{2} + 2x \Big|_{-1}^{2} = \frac{-8}{3} + \frac{4}{2} + 4 - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} - 2\right)$$

$$= \frac{-8}{3} + 2 + 4 - \frac{1}{3} - \frac{1}{2} + 2 = 8 - \cancel{\frac{9}{3}} - \frac{1}{2} = 5 - \frac{1}{2} = \boxed{\frac{9}{2}}$$

3. [35 points]

(a) Compute $f'(x)$ when $f(x) = e^x + \frac{1}{e^x} + x^e + \frac{1}{x^e} + e^e + \frac{1}{e^e} + \frac{x}{e} + \frac{e}{x} + ex + \frac{1}{ex}$.

$$= e^x + e^{-x} + x^e + x^{-e} + \underbrace{e^e + \frac{1}{e^e}}_{\text{constant}} + \frac{1}{e}x + ex^{-1} + ex + \frac{1}{e}x^{-1}$$

$$f'(x) = \boxed{e^x - e^{-x} + ex^{e-1} - ex^{-e-1} + 0 + 0 + \frac{1}{e} - ex^{-2} + e - \frac{1}{e}x^{-2}}$$

(b) Consider $f(x) = e^{\sin x}$. Compute $f'(0)$. Simplify.

$$f'(x) = e^{\sin x}\cos x$$

$$f'(0) = e^{\cancel{\sin 0}^{\,0}} \cdot \cancel{\cos 0}^{\,1}$$

$$= \cancel{e^0}^{\,1} \cdot 1$$

$$= \boxed{1}$$

3. (Continued)

(c) Consider $f(x) = \dfrac{1}{\tan\left(e^x + \sqrt{x}\right)} + \dfrac{1}{e^{\tan\sqrt{x}}}$. Compute $f'(x)$.

$$= \left[\tan(e^x + \sqrt{x})\right]^{-1} + e^{-\tan\sqrt{x}}$$

$$f'(x) = \boxed{-\left[\tan(e^x+\sqrt{x})\right]^{-2} \cdot \sec^2(e^x+\sqrt{x}) \cdot \left(e^x + \frac{1}{2\sqrt{x}}\right) + e^{-\tan\sqrt{x}}\left(-\sec^2\sqrt{x}\right) \cdot \frac{1}{2\sqrt{x}}}$$

(d) Consider $f(x) = x^7 + 7^x$. Compute $f'(x)$.

$$f'(x) = 7x^6 +$$

Recall: $\frac{d}{dx} 7^x = 7^x \left[\lim_{h \to 0} \frac{7^h - 1}{h}\right]$