Name: Answer Key

Math 106 Quiz #7 March 19, 2018

- This is a closed-book quiz. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

1. [20 points] Evaluate each of the following integrals. Simplify answers. Justify all steps.

(a) $\int \frac{x^6}{\sqrt{x^7+5}}\,dx = \frac{1}{7}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{7}\int u^{-1/2}\,du = \frac{1}{7}\left[\frac{u^{1/2}}{1/2}\right] + C$

$\boxed{\begin{array}{l} u = x^7+5 \\ du = 7x^6\,dx \\ \frac{1}{7}du = x^6\,dx \end{array}}$

$= \frac{2}{7}\sqrt{u} + C = \boxed{\frac{2}{7}\sqrt{x^7+5} + C}$

(b) $\int_1^7 \sqrt{2x+2}\,dx = \frac{1}{2}\int_4^{16} \sqrt{u}\,du = \frac{1}{2}\int_4^{16} u^{1/2}\,du = \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]\Bigg|_4^{16}$

$\boxed{\begin{array}{l} u = 2x+2 \\ du = 2dx \\ \frac{1}{2}du = dx \end{array}}$ $\boxed{\begin{array}{l} x=1 \Rightarrow u = 2+2=4 \\ x=7 \Rightarrow u = 14+2=16 \end{array}}$

$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}u^{3/2}\Big|_4^{16} = \frac{1}{3}u^{3/2}\Big|_4^{16}$

$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$

$4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$

$= \frac{1}{3}\left[16^{3/2} - 4^{3/2}\right] = \frac{1}{3}[64-8] = \boxed{\frac{56}{3}}$

(c) $\int \frac{\sec^2 x}{(1+\tan x)^{\frac{9}{7}}}\,dx = \int \frac{1}{u^{9/7}}\,du = \int u^{-9/7}\,du = \frac{u^{-2/7}}{-\frac{2}{7}} + C$

$\boxed{\begin{array}{l} u = 1+\tan x \\ du = \sec^2 x\,dx \end{array}}$

$= -\frac{7}{2}u^{-2/7} + C = \boxed{\frac{-7}{2(1+\tan x)^{2/7}} + C}$

1. (continued) Evaluate each of the following integrals. Simplify answers. Justify all steps.

(d) $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \sin^3(2x)\cos(2x)\, dx = \frac{1}{2}\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} u^3 du = \frac{1}{2}\left[\frac{u^4}{4}\right]\Big|_{1/2}^{\sqrt{3}/2} = \frac{1}{8}\, u^4 \Big|_{1/2}^{\sqrt{3}/2}$

$u = \sin(2x)$

$du = 2\cos(2x)dx$

$\frac{1}{2} du = \cos(2x)dx$

$x = \frac{\pi}{12} \Rightarrow u = \sin \frac{\pi}{6} = \frac{1}{2}$

$x = \frac{\pi}{6} \Rightarrow u = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$= \frac{1}{8}\left[\left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{2}\right)^4\right]$

$= \frac{1}{8}\left[\frac{9}{16} - \frac{1}{16}\right] = \frac{1}{\cancel{8}}\left[\frac{\cancel{8}}{16}\right]$

$= \boxed{\frac{1}{16}}$

$(\sqrt{3})^4 = \left[(\sqrt{3})^2\right]^2 = 3^2 = 9$

$2^4 = 16$

(e) $\int_1^2 \frac{1}{x^2\left(3+\frac{1}{x}\right)^2}\, dx = -\int_4^{7/2} \frac{1}{u^2} du = -\int_4^{7/2} u^{-2} du = \cancel{-}\frac{u^{-1}}{\cancel{-}1}\Big|_4^{7/2}$

$u = 3 + \frac{1}{x}$

$du = \frac{-1}{x^2}dx$

$-du = \frac{1}{x^2}dx$

$x = 1 \Rightarrow u = 3+1 = 4$

$x = 2 \Rightarrow u = 3 + \frac{1}{2} = \frac{7}{2}$

$= \frac{1}{u}\Big|_4^{7/2} = \frac{1}{(7/2)} - \frac{1}{4}$

$= \frac{2}{7} - \frac{1}{4}$

$= \frac{8}{28} - \frac{7}{28} = \boxed{\frac{1}{28}}$

1. (continued) Evaluate each of the following integrals. Simplify answers. Justify all steps.

(f) $\displaystyle\int \frac{\sin(5+\sqrt{x})}{\sqrt{x}}\,dx = 2\int \sin u\,du = -2\cos u + C$

$u = 5+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$= \boxed{-2\cos(5+\sqrt{x})+C}$

(g) $\displaystyle\int x(x+1)^8\,dx = \int (u-1)u^8\,du = \int u^9 - u^8\,du$

$u = x+1 \Rightarrow x = u-1$

$du = dx$

$= \frac{u^{10}}{10} - \frac{u^9}{9} + C$

$= \boxed{\frac{(x+1)^{10}}{10} - \frac{(x+1)^9}{9} + C}$