Answer Key

Math 106 Quiz #6 March 6, 2017

1. [20 points] Evaluate each of the following definite integrals.

(a) $\displaystyle\int_{-1}^{2} x^2 - 3x\ dx = \frac{x^3}{3} - \frac{3x^2}{2}\bigg|_{-1}^{2} = \frac{8}{3} - 6 - \left(\frac{(-1)^3}{3} - \frac{3}{2}\right) = \frac{8}{3} - 6 + \frac{1}{3} + \frac{3}{2}$

$\displaystyle = \frac{9}{3} - 6 + \frac{3}{2} = 3 - 6 + \frac{3}{2} = -3 + \frac{3}{2} = -\frac{6}{2} + \frac{3}{2} = \boxed{-\frac{3}{2}}$

(b) $\displaystyle\int_{-7}^{7} 7\ dx = 7x\bigg|_{-7}^{7} = 49 - (-49) = \boxed{98}$

(c) $\displaystyle\int_{-2}^{2} 2x\ dx = x^2\bigg|_{-2}^{2} = 4 - 4 = \boxed{0}$

(d) $\displaystyle\int_{0}^{4} (4-x)\sqrt{x}\ dx = \int_{0}^{4} 4\sqrt{x} - x^{\frac{3}{2}}\ dx = 4\left(\frac{2}{3}\right)x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\bigg|_{0}^{4}$

$\displaystyle = \left(\frac{8}{3}\right)4^{\frac{3}{2}} - \left(\frac{2}{5}\right)4^{\frac{5}{2}} - (0-0) = \left(\frac{8}{3}\right)\left(\sqrt{4}\right)^3 - \left(\frac{2}{5}\right)\left(\sqrt{4}\right)^5 = \left(\frac{8}{3}\right)(2)^3 - \left(\frac{2}{5}\right)(2)^5$

$\displaystyle = \frac{64}{3} - \frac{64}{5} = \frac{320}{15} - \frac{192}{15} = \boxed{\frac{128}{15}}$

(e) $\displaystyle\int_{0}^{1} (x+2)(x-3)\ dx = \int_{0}^{1} x^2 - x - 6\ dx = \frac{x^3}{3} - \frac{x^2}{2} - 6x\bigg|_{0}^{1}$

$\displaystyle = \frac{1}{3} - \frac{1}{2} - 6 - (0 - 0 - 0) = \frac{2}{6} - \frac{3}{6} - \frac{36}{6} = \boxed{-\frac{37}{6}}$

(f) $\displaystyle\int_{1}^{4} \frac{1+x}{\sqrt{x}}\ dx = \int_{1}^{4} \frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}}\ dx = \int_{1}^{4} x^{-\frac{1}{2}} + x^{\frac{1}{2}}\ dx = 2x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}}\bigg|_{1}^{4}$

$\displaystyle = 2\sqrt{4} + \left(\frac{2}{3}\right)4^{\frac{3}{2}} - \left(2 + \frac{2}{3}\right) = 4 + \frac{16}{3} - 2 - \frac{2}{3} = 2 + \frac{14}{3} = \frac{6}{3} + \frac{14}{3} = \boxed{\frac{20}{3}}$

(g) $\displaystyle\int_{\frac{\pi}{6}}^{\pi} \sin\theta\ d\theta = -\cos\theta\bigg|_{\frac{\pi}{6}}^{\pi} = -\cos\pi - \left(-\cos\left(\frac{\pi}{6}\right)\right) = -(-1) + \frac{\sqrt{3}}{2} = \boxed{1 + \frac{\sqrt{3}}{2}}$

(h) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x\ dx = \sin x\bigg|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \sin\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{1}{2} = \boxed{\frac{\sqrt{3}-1}{2}}$

(i) $\displaystyle\int_0^{\frac{\pi}{3}} \sec x \tan x + \sec^2 x \ dx = \sec x + \tan x \Big|_0^{\frac{\pi}{3}} = \sec\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{3}\right) - (\sec 0 + \tan 0)$

$= 2 + \sqrt{3} - (1 + 0) = \boxed{1 + \sqrt{3}}$

(j) $\displaystyle\int_1^2 \frac{(1+x)^2}{x^4} \ dx = \int_1^2 \frac{1 + 2x + x^2}{x^4} \ dx = \int_1^2 \frac{1}{x^4} + \frac{2x}{x^4} + \frac{x^2}{x^4} \ dx$

$\displaystyle= \int_1^2 x^{-4} + 2x^{-3} + x^{-2} \ dx = \frac{x^{-3}}{-3} + 2\left(\frac{x^{-2}}{-2}\right) + \frac{x^{-1}}{-1}\Big|_1^2$

$\displaystyle= -\frac{1}{3x^3} - \frac{1}{x^2} - \frac{1}{x}\Big|_1^2 = -\frac{1}{24} - \frac{1}{4} - \frac{1}{2} - \left(-\frac{1}{3} - 1 - 1\right) = -\frac{1}{24} - \frac{1}{4} - \frac{1}{2} + \frac{1}{3} + 2$

$\displaystyle= -\frac{1}{24} - \frac{6}{24} - \frac{12}{24} + \frac{8}{24} + \frac{48}{24} = \boxed{\frac{37}{24}}$