Name: Me

Math 106 Quiz #3 February 13, 2017

- This is a closed-book quiz. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.
- Please *show* all of your work and *justify* all of your answers.

1. [30 points] Compute the most general antiderivative for each of the following functions.

(a) $4x^7+\frac{7}{x^4}+x^{\frac{4}{7}}+\frac{1}{x^{\frac{7}{4}}} = 4x^7+7x^{-4}+x^{4/7}+x^{-7/4}$

antiderivative $\frac{4x^8}{8}+\frac{7x^{-3}}{-3}+\frac{x^{11/7}}{11/7}+\frac{x^{-3/4}}{-3/4}+C = \boxed{\frac{x^8}{2}-\frac{7}{3x^3}+\frac{7}{11}x^{11/7}-\frac{4}{3}x^{-3/4}+C}$

(b) $(x+1)(x+2) \overset{\text{FOIL}}{=} x^2+3x+2$

antiderivative $\boxed{\frac{x^3}{3}+\frac{3x^2}{2}+2x+C}$

(c) $\frac{\left(x^2+\frac{1}{x}\right)\left(x+\frac{1}{x^2}\right)}{\sqrt{x}} \overset{\text{FOIL}}{=} \frac{x^3+\overset{1}{\cancel{x^2\left(\frac{1}{x^2}\right)}}+\overset{1}{\cancel{\frac{1}{x}}\cdot\cancel{x}}+\overset{\frac{1}{x^3}}{\frac{1}{x}\cdot\frac{1}{x^2}}}{\sqrt{x}} = \frac{x^3+2+x^{-3}}{x^{1/2}} \overset{\text{DIVIDE}}{=} x^{5/2}+2x^{-1/2}+x^{-7/2}$

antiderivative $\frac{2}{7}x^{7/2}+\frac{2x^{1/2}}{1/2}+\frac{x^{-5/2}}{-5/2}+C = \boxed{\frac{2}{7}x^{7/2}+4\sqrt{x}-\frac{2}{5}x^{-5/2}+C}$

(d) $\sec^2 x+2\sin x-\cos x-\sec x\tan x$

antiderivative

$\boxed{\tan x-2\cos x-\sin x-\sec x+C}$

(e) $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)(1+x) \overset{\text{FOIL}}{=} \sqrt{x}+x\sqrt{x}+\frac{1}{\sqrt{x}}+\frac{x}{\sqrt{x}} = \underbrace{x^{1/2}+x^{3/2}+x^{-1/2}+x^{1/2}}_{\text{2 copies}} = 2x^{1/2}+x^{3/2}+x^{-1/2}$

antiderivative $2\cdot\frac{x^{3/2}}{3/2}+\frac{x^{5/2}}{5/2}+\frac{x^{1/2}}{1/2} = \boxed{\frac{4}{3}x^{3/2}+\frac{2}{5}x^{5/2}+2\sqrt{x}+C}$

2. [10 points] Consider a function f with $f''(x) = -3+12x-12x^2$ and that satisfies $f'(1) = -4$ and $f(0) = 4$.

(a) Find $f(x)$.

$f'(x) = -3x + \frac{12x^2}{2} - \frac{12x^3}{3} + C_1 = -3x+6x^2-4x^3+C_1$

Test $f'(1) = \underbrace{-3+6-4}_{-1} + C_1 \overset{set}{=} -4$ Solve $-1+C_1=-4 \Rightarrow C_1=-3$

Clean up $f'(x) = -3x+6x^2-4x^3-3$

$\hookrightarrow$ $f(x) = \frac{-3}{2}x^2 + \frac{6x^3}{3} - \frac{4x^4}{4} - 3x + C_2 = \frac{-3}{2}x^2 + 2x^3 - x^4 - 3x + C_2$

Test $f(0) = 0+0-0-0+C_2 \overset{set}{=} 4 \Rightarrow C_2 = 4$

(b) Compute $f(1)$. Simplify.

Finally, $f(x) = \boxed{\frac{-3}{2}x^2 + 2x^3 - x^4 - 3x + 4}$

$f(1) = \frac{-3}{2} + \underbrace{2-1}_{1} \underbrace{-3+4}_{1} = \frac{-3}{2} + 2 = \frac{-3}{2} + \frac{4}{2} = \boxed{\frac{1}{2}}$

3. [10 points] Consider a function f with $f'(x) = \sec^2 x - 4\sin x$ and that satisfies $f(\pi) = -6$.

(a) Find $f(x)$.

$\hookrightarrow$ $f(x) = \tan x + 4\cos x + C$

Test $f(\pi) = \underbrace{\tan\pi}_{0} + 4\underbrace{\cos\pi}_{-1} + C \overset{set}{=} -6$

$-4 + C = -6$

$\Rightarrow C = -2$

$f(x) = \tan x + 4\cos x - 2$

(b) Compute $f\left(\frac{\pi}{3}\right)$. Simplify.

$f(\pi/3) = \underbrace{\tan\pi/3}_{\sqrt{3}} + 4\underbrace{\cos\pi/3}_{1/2} - 2 = \sqrt{3} + \cancel{4(1/2)}^{2} \cancel{-2} = \boxed{\sqrt{3}}$

Cancel