ANSWER KEY

Math 106 | Quiz #2 | February 6, 2017

1. [4 points] Differentiate each of the following functions. Do **not** simplify your answer.

(a) $f(x) = \sqrt{\cos\left(\frac{1}{x}\right)}$

$f'(x) = \boxed{\frac{1}{2\sqrt{\cos\left(\frac{1}{x}\right)}}\left(-\sin\left(\frac{1}{x}\right)\right)\left(-\frac{1}{x^2}\right)}$

(b) $f(x) = (\sec x + 1)\sin x$

$f'(x) = \boxed{(\sec x + 1)\cos x + \sin x(\sec x \tan x)}$

OR simplify first

$f(x) = (\sec x + 1)\sin x = \sec x \sin x + \sin x = \frac{1}{\cos x}\sin x + \sin x = \frac{\sin x}{\cos x} + \sin x = \tan x + \sin x$

so $f'(x) = \boxed{\sec^2 x + \cos x}$

NOTE: These are equal solutions because the first solution simplifies to

$f'(x) = (\sec x + 1)\cos x + \sin x(\sec x \tan x) = \sec x \cos x + \cos x + \sin x \sec x \tan x$

$= \frac{1}{\cos x}\cos x + \cos x + \sin x \frac{1}{\cos x}\tan x$

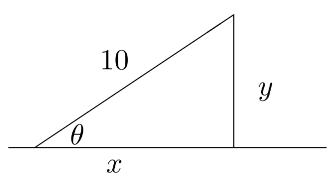
$= 1 + \cos x + \frac{\sin x}{\cos x}\tan x = 1 + \cos x + \tan^2 x = (1 + \tan^2 x) + \cos x = \sec^2 x + \cos x$

2. [5 points] Consider $f(x) = \frac{\sin x}{1 + \cos x}$. Compute $f'(x)$. **Simplify** your answer.

$$f'(x) = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{\cos x + 1}{(1+\cos x)^2} = \boxed{\frac{1}{1+\cos x}}$$

3. [6 points] The top of a ten foot ladder is sliding down a vertical wall at the rate of one foot every second. Consider the angle formed by the bottom of the ladder and the ground. How fast is this angle changing when the top of the ladder is five feet above the ground?

- Diagram



• Variables

Let $x =$ distance between bottom of ladder and wall at time t
Let $y =$ distance between top of ladder and ground at time t
Let $\theta =$ angle formed by the ground and base of ladder at time t

Find $\dfrac{d\theta}{dt} =?$ when $y = 5$ ft

and $\dfrac{dy}{dt} = -1 \dfrac{\text{ft}}{\text{sec}}$

• Equation relating the variables:

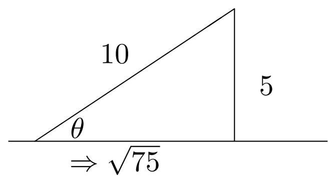
We have $\sin\theta = \dfrac{y}{10}$.

• Differentiate both sides w.r.t. time t.

$\dfrac{d}{dt}(\sin\theta) = \dfrac{d}{dt}\left(\dfrac{y}{10}\right) \implies \cos\theta \dfrac{d\theta}{dt} = \dfrac{1}{10}\dfrac{dy}{dt}$(Related Rates!)

• Extra Solvable Information

We're not given θ for this problem, but we can still compute $\cos\theta$ from trig. relations on the diagram's triangle with $\cos\theta = \frac{\text{adj}}{\text{hyp}}$. At the key momeny, when $y = 5$, we can use the Pyth. Theorem to compute $x = \sqrt{(10)^2 - (5)^2} = \sqrt{75}$. Finally, $\cos\theta = \dfrac{\sqrt{75}}{10}$.



• Substitute Key Moment Information (now and not before now!!!):

$\cos\theta \dfrac{d\theta}{dt} = \dfrac{1}{10}\dfrac{dy}{dt}$

$\dfrac{\sqrt{75}}{10}\dfrac{d\theta}{dt} = \dfrac{1}{10}(-1)$

• Solve for the desired quantity:

$\dfrac{d\theta}{dt} = -\dfrac{1}{10} \cdot \dfrac{10}{\sqrt{75}} = -\dfrac{1}{\sqrt{75}} \dfrac{\text{rad}}{\text{sec}}$

Note: $\sqrt{75} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

• Answer the question that was asked: The angle is decreasing at a rate of $\dfrac{1}{\sqrt{75}}$ radians every second at this moment.

4. [5 points] Let $f(x) = \sin x + \cos(2x)$. Compute $f'\left(\frac{\pi}{6}\right)$. Simplify your answer completely.

$f'(x) = \cos x - 2\sin(2x)$

$f'\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) - 2\sin\left(2\left(\frac{\pi}{6}\right)\right) = \cos\left(\frac{\pi}{6}\right) - 2\sin\left(\frac{\pi}{3}\right)$

$= \dfrac{\sqrt{3}}{2} - 2\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{2} - \sqrt{3} = \boxed{-\dfrac{\sqrt{3}}{2}}$