Name:_____

Amherst College

DEPARTMENT OF MATHEMATICS

Math 106

Midterm Exam #3

April 18, 2018

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $\ln(e^3)$, $e^{2\ln 3}$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		28
2		12
3		8
4		40
5		6
6		6
Total		100

1. [28 Points] Compute each of the following derivatives.

(a) y' where $y = \ln\left(\dfrac{\ln x\ \sqrt{1+e^x}}{(4-x^6)^3\ e^{-\cos x}}\right)$ Do not simplify your final answer here.

(b) $\dfrac{d}{dx}\ (\cos x)^{\sin x}$

1. (Continued) Compute each of the following derivatives.

(c) $\dfrac{dy}{dx}$ where $y = \ln\left(\ln\left(\ln\left(e^x\right)\right)\right) + \dfrac{e}{\ln x} + \dfrac{\ln x}{e} + \left(\ln x \cdot e^x\right) + \ln\left(xe^x\right)$

(d) $\dfrac{dy}{dx}$ where $y = e^{\ln(\ln x)} + \ln\left(e^5\right) + e^{\ln x} + 5^{\ln e}$

2. [12 Points]

(a) Find the Absolute Maximum and/or Minimum **Values** for the function $f(x) = \dfrac{x+2}{e^x}$.

2. (Continued)

(b) At what point on the curve $y = [\ln(x+4)]^2$ is the tangent line horizontal?

3. [8 Points] Sketch the graphs for each of the following functions. For each, state both the Domain and the Range.

(a) $f(x) = e^x$

(b) $f(x) = \ln x$

4. [40 Points] Evaluate each of the following integrals. Simplify.

(a) $\displaystyle\int \frac{1}{x^3\, e^{\frac{1}{x^2}}}\, dx$

(b) $\displaystyle\int_1^{\sqrt{6}} \frac{x}{7-x^2}\, dx$

4. (Continued) Evaluate each of the following integrals. Simplify.

(c) $\displaystyle\int_{e^3}^{e^8} \frac{4}{x\sqrt{1+\ln x}}\,dx$

(d) $\displaystyle\int_{-3}^{-1} \frac{1-x}{x^2}\,dx$

4. (Continued) Evaluate the following integral. Simplify.

(e) Show that $\displaystyle\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \tan(3x)\, dx = \boxed{\frac{\ln 3}{6}}$

4. (Continued) Evaluate the following integral. Simplify.

(f) Show that $\displaystyle\int_0^{\ln 2} \left(e^x + \frac{1}{e^{2x}}\right)^2 \, dx = \boxed{\frac{175}{64}}$

5. [6 Points] Find the function $f(x)$ that satsifies $f'(x) = \dfrac{1}{e^{2x}\left(1-2e^{-2x}\right)^2}$ and $f(0) = -1$.

6. [6 Points] Consider $y = \ln x$. **Prove** that $\dfrac{dy}{dx} = \dfrac{1}{x}$.