Name:_____

Amherst College

DEPARTMENT OF MATHEMATICS

Math 106

Midterm Exam #2

March 26, 2018

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		18
2		4
3		32
4		32
5		4
6		10
Total		100

1. [18 Points] Compute $\int_{-1}^{2} 2 - 3x - x^2 \, dx$ using two different methods:

(a) Fundamental Theorem of Calculus

(b) Limit Definition of the Definite Integral.

2. [4 Points] Compute $g'(x)$ where $g(x) = \displaystyle\int_x^3 \frac{\sec^2 t}{\sqrt{t^2 + 9\sin t}}\, dt$.

3. [32 Points] Evaluate each of the following integrals. Simplify.

(a) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 4\sin x \cdot \cos^3 x \; dx$

(b) $\displaystyle\int \frac{1}{\sqrt{x}\,\sqrt{2+\sqrt{x}}}\; dx$

3. (Continued) Evaluate each of the following integrals. Simplify.

(c) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan^3 x}\, dx$

(d) $\displaystyle\int \frac{\sec\left(9+\frac{1}{x^2}\right)\tan\left(9+\frac{1}{x^2}\right)}{x^3}\, dx$

4. [32 Points] Evaluate each of the following integrals. Simplify.

(a) $\displaystyle\int_1^4 \frac{1-x}{\sqrt{x}}\,dx$

(b) $\displaystyle\int_{-2}^{-1} \left(x-\frac{1}{x}\right)^2\,dx$

4. (Continued) Evaluate each of the following integrals. Simplify.

(c) $\int \sqrt{x}\cos\left(x\sqrt{x}\right)\ dx$

(d) $\int_{-3}^{-2} x(x+2)^7\ dx$

5. [4 Points] Compute $f(x)$ where $f'(x) = \sin\left(\frac{x}{2}\right)$ and $f\left(-\frac{\pi}{3}\right) = 4\sqrt{3}$

6. [10 Points] Consider an object travelling with velocity $v(t) = 1 - t$ meters per second.

(a) Sketch and label both $v(t)$ and $|v(t)|$.

(b) Compute the **Total Distance** travelled by the object from time $t = 0$ to $t = 3$.

(c) Compute the Total Distance Integral from part (b) (this time) using Area Interpretations. Your answers from (b) and (c) should both match.