Math 106 HW #14 Review Packet Exam #2

1. $$\int_0^2 x^2-7x+3\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$$

Here $f(x) = x^2-7x+3$

$a=0 \quad b=2$

$\Delta x = \frac{b-a}{n} = \frac{2}{n}$

$x_i = a + i\Delta x$

$= 2i/n$

$$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^2 - 7\left(\frac{2i}{n}\right)+3\right]$$

$$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\frac{4i^2}{n^2}-\frac{14i}{n}+3$$

$$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\frac{4i^2}{n^2} - \frac{2}{n}\sum_{i=1}^{n}\frac{14i}{n} + \frac{2}{n}\sum_{i=1}^{n}3$$

$$= \lim_{n\to\infty}\frac{8}{n^3}\sum_{i=1}^{n}i^2 - \frac{28}{n^2}\sum_{i=1}^{n}i + \frac{6}{n}\cancelto{n}{\sum_{i=1}^{n}1}$$

$$= \lim_{n\to\infty}\frac{8}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] - \frac{28}{n^2}\left[\frac{n(n+1)}{2}\right]+6$$

$$= \lim_{n\to\infty}\frac{4}{3}\left(\cancel{\frac{n}{n}}\right)\cancelto{1}{\underbrace{\left(\frac{n+1}{n}\right)}_{1+1/n}}\cancelto{\boxed{2}}{\underbrace{\left(\frac{2n+1}{n}\right)}_{2+1/n}} - 14\left(\cancel{\frac{n}{n}}\right)\cancelto{1}{\underbrace{\left(\frac{n+1}{n}\right)}_{1+1/n}}+6$$

$$= \frac{8}{3}-14+6$$

$$= 8/3 - 8 = \frac{8}{3}-\frac{24}{3} = \boxed{\frac{-16}{3}}$$

Not Required But
Free Double Check: FTC

$$\int_0^2 x^2-7x+3\,dx = \frac{x^3}{3}-\frac{7x^2}{2}+3x\Big|_0^2 = \frac{8}{3}-14+6-(0-0+0) = \frac{8}{3}-8 = \boxed{-16/3}$$

2. $\displaystyle\int_1^4 9x-x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} f(1+3i/n)\left[\frac{3}{n}\right]$

Here $f(x)=9x-x^2$

$a=1,\ b=4$

$\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$

$x_i = a+i\Delta x$

$= 1+3i/n$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 9(1+3i/n) - \underbrace{(1+3i/n)^2}_{1+6i/n+\frac{9i^2}{n^2}}$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 9+\frac{27i}{n}-1-\frac{6i}{n}-\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 8+\frac{21i}{n}-\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 8 + \frac{3}{n}\sum_{i=1}^{n}\frac{21i}{n} - \frac{3}{n}\sum_{i=1}^{n}\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\cancel{\frac{24}{n}\sum_{i=1}^{n}1}^{\,n} + \frac{63}{n^2}\sum_{i=1}^{n} i - \frac{27}{n^3}\sum_{i=1}^{n} i^2$$

$$= \lim_{n\to\infty} 24 + \frac{63}{n^2}\left[\frac{n(n+1)}{2}\right] - \frac{27}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} 24 + \frac{63}{2}\left(\cancel{\frac{n}{n}}\right)\cancel{\underbrace{\left(\frac{n+1}{n}\right)}_{1+1/n}}^{\,1} - \frac{27}{6}\left(\cancel{\frac{n}{n}}\right)\cancel{\underbrace{\left(\frac{n+1}{n}\right)}_{1+1/n}}^{\,1}\cancel{\underbrace{\left(\frac{2n+1}{n}\right)}_{2+1/n}}^{\,2}$$

$$= 24 + \frac{63}{2} - \cancel{\frac{54}{6}}^{\,9}$$

$$= 15 + \frac{63}{2} = \frac{30}{2}+\frac{63}{2} = \boxed{\frac{93}{2}}$$

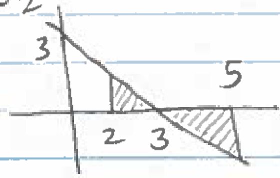
Not Required But

Free Double Check: FTC

$$\int_1^4 9x-x^2\,dx = \frac{9x^2}{2} - \frac{x^3}{3}\Big|_1^4 = 72-\frac{64}{3}-\left(\frac{9}{2}-\frac{1}{3}\right) = 72-\frac{9}{2}-\frac{63}{3} = 72-\frac{9}{2}-21$$

$$= 51 - 9/2 = \frac{102}{2}-\frac{9}{2} = \boxed{\frac{93}{2}}$$

3. $\int_2^5 3-x\,dx$

(a) 

$\frac{1}{2}bh - \frac{1}{2}bh$

$\frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = \frac{1}{2} - 2 = \boxed{\frac{-3}{2}}$

(b) $\int_2^5 3-x\,dx = 3x - \frac{x^2}{2}\Big|_2^5 = 15 - \frac{25}{2} - (\underbrace{6-2}_{4}) = 15 - \frac{25}{2} - 4 = 11 - \frac{25}{2} = \frac{22}{2} - \frac{25}{2}$

$= \boxed{\frac{-3}{2}}$

(c) $\int_2^5 3-x\,dx \quad = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(2+\frac{3i}{n}\right)\left(\frac{3}{n}\right)$

Here $f(x) = 3-x$

$a = 2,\ b = 5$

$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}$

$x_i = a + i\Delta x = 2 + \frac{3i}{n}$

$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} 3 - (2 + 3i/n)$

$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} 1 - 3i/n$

$= \lim_{n\to\infty} \cancel{\frac{3}{n} \sum_{i=1}^{n} 1}^{\,n} - \frac{3}{n} \sum_{i=1}^{n} 3i/n$

$= \lim_{n\to\infty} 3 - \frac{9}{n^2} \sum_{i=1}^{n} i$

$= \lim_{n\to\infty} 3 - \frac{9}{n^2}\left[\frac{n(n+1)}{2}\right]$

$= \lim_{n\to\infty} 3 - \frac{9}{2}\cancel{\left(\frac{n}{n}\right)}\cancel{\underbrace{\left(\frac{n+1}{n}\right)}_{1+1/n}}^{\,1}$

$= 3 - \frac{9}{2} = \frac{6}{2} - \frac{9}{2} = \boxed{\frac{-3}{2}}$ Match!

4. $\frac{d}{dx}\int_{x}^{7} 1-\sin t\,dt = \frac{d}{dx}\left[-\int_{7}^{x} 1-\sin t\,dt\right] = \boxed{-(1-\sin x)}$

5. $g(x) = \int_{3x}^{7} 7t^2+\sin t\ dt$

$g'(x) = \frac{d}{dx}\int_{3x}^{7} 7t^2+\sin t\ dt = \frac{d}{dx}\left[-\int_{7}^{3x} 7t^2+\sin t\ dt\right]$

$= -\left[7(3x)^2+\sin(3x)\right](3) = -3\left[63x^2+\sin(3x)\right]$

Chain Rule here

$g''(x) = \boxed{-3\left[126x+3\cos(3x)\right]}$

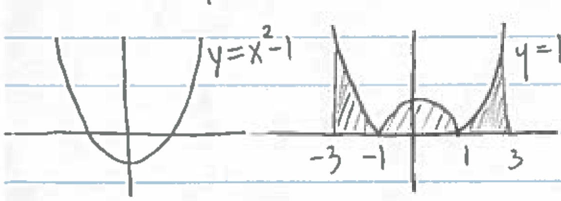
6. $\int \frac{1}{\sqrt[3]{(7-5z)^2}}\,dz = \int \frac{1}{(7-5z)^{2/3}}\,dz = -\frac{1}{5}\int \frac{1}{u^{2/3}}\,du = -\frac{1}{5}\int u^{-2/3}\,du$

$\boxed{\begin{array}{c} u = 7-5z \\ du = -5\,dz \\ -\frac{1}{5}du = dz \end{array}}$

$= -\frac{1}{5}(3)u^{1/3}+C = \boxed{-\frac{3}{5}(7-5z)^{1/3}+C}$

7. $\int_{-3}^{3}|x^2-1|\,dx = \int_{-3}^{-1}\overset{x^2-1}{|x^2-1|}\,dx + \int_{-1}^{1}\overset{1-x^2}{|x^2-1|}\,dx + \int_{1}^{3}\overset{x^2-1}{|x^2-1|}\,dx$

$|x^2-1| = \begin{cases} x^2-1 & \text{if } x\geq 1, x\leq -1 \\ -(x^2-1) & \text{if } -1<x<1 \end{cases}$



$= \int_{-3}^{-1} x^2-1\,dx + \int_{-1}^{1} 1-x^2\,dx + \int_{1}^{3} x^2-1\,dx$

$= \frac{x^3}{3}-x\Big|_{-3}^{-1} + x-\frac{x^3}{3}\Big|_{-1}^{1} + \frac{x^3}{3}-x\Big|_{1}^{3}$

$= -\frac{1}{3}+1-(-9+3)+\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)+9-3-\left(\frac{1}{3}-1\right)$

$= -\cancel{\frac{1}{3}}+\cancel{1}+6+1-\frac{1}{3}+1-\frac{1}{3}+6+\frac{2}{3}$

$= 14+\frac{2}{3} = \frac{42}{3}+\frac{2}{3} = \boxed{\frac{44}{3}}$

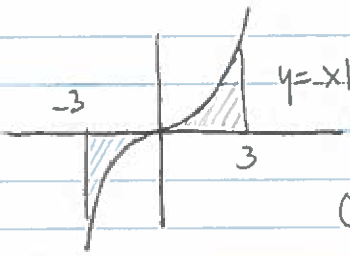
8. $$\int_{-3}^{3} x|x|\,dx = \int_{-3}^{0} x\overset{(-x)}{\cancel{|x|}}\,dx + \int_{0}^{3} x\overset{x}{\cancel{|x|}}\,dx = \int_{-3}^{0} -x^2\,dx + \int_{0}^{3} x^2\,dx$$

Recall $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$$= -\frac{x^3}{3}\Big|_{-3}^{0} + \frac{x^3}{3}\Big|_{0}^{3}$$

$$= 0 - \left(-\frac{(-3)^3}{3}\right) + \frac{3^3}{3} - 0$$

$$= -9 + 9 = \boxed{0}$$



$y = x|x| = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$

ODD function

9. $$\int_{0}^{\pi/8} \tan^3(2x)\sec^2(2x)\,dx = \frac{1}{2}\int_{0}^{1} u^3\,du = \frac{1}{2}\left(\frac{u^4}{4}\right)\Big|_{0}^{1} = \frac{1}{8} - 0 = \boxed{\frac{1}{8}}$$

$u = \tan(2x)$
$du = \sec^2(2x)(2)\,dx$
$\frac{1}{2}du = \sec^2(2x)\,dx$

$x = 0 \Rightarrow u = \tan 0 = 0$
$x = \pi/8 \Rightarrow u = \tan(\pi/4) = 1$

10. $$\int_{0}^{\pi/3} \frac{3\sin x\cos x}{(1+3\sin^2 x)^2}\,dx = \frac{1}{2}\int_{1}^{13/4} \frac{1}{u^2}\,du = \frac{1}{2}\int_{1}^{13/4} u^{-2}\,du = \frac{1}{2}\,\frac{u^{-1}}{(-1)}\Big|_{1}^{13/4} = \frac{-1}{2u}\Big|_{1}^{13/4}$$

$u = 1+3\sin^2 x$
$du = 6\sin x\cdot\cos x\,dx$
$\frac{1}{2}du = 3\sin x\cdot\cos x\,dx$

$x = 0 \Rightarrow u = 1 + 3\overset{0}{\cancel{\sin^2 0}} = 1$
$x = \pi/3 \Rightarrow u = 1 + 3\sin^2(\pi/3)$
$= 1 + 3\left(\sqrt{3}/2\right)^2$
$= 1 + 9/4 = 13/4$

$$= \frac{-1}{2(13/4)} - \left(-\frac{1}{2}\right)$$

$$= -\frac{2}{13} + \frac{1}{2}$$

$$= -\frac{4}{26} + \frac{13}{26} = \boxed{\frac{9}{26}}$$

11. $\int 7\cos(5x) - 5\sin(7x)\,dx = 7\int \cos(5x)\,dx - 5\int \sin(7x)\,dx$

$\boxed{\begin{array}{c} u=5x \\ du=5dx \\ \frac{1}{5}du=dx \end{array}}$

$= \frac{7}{5}\int \cos u\,du - \frac{5}{7}\int \sin w\,dw$

$\boxed{\begin{array}{c} w=7x \\ dw=7dx \\ \frac{1}{7}dw=dx \end{array}}$

$= \frac{7}{5}\sin u \quad + \frac{5}{7}\cos w + C$

$= \boxed{\frac{7}{5}\sin(5x) + \frac{5}{7}\cos(7x) + C}$

12. $\int (x^{7/2} + x^{-1/3})\sqrt{x}\,dx = \int x^4 + x^{1/6}\,dx = \boxed{\frac{x^5}{5} + \frac{6}{7}x^{7/6} + C}$

$-\frac{1}{3} + \frac{1}{2} = -\frac{2}{6} + \frac{3}{6} = \frac{1}{6}$

13. $\int x(x+1)^{14}\,dx \; = \int (u-1)\,u^{14}\,du = \int u^{15} - u^{14}\,du = \frac{u^{16}}{16} - \frac{u^{15}}{15} + C$

$\boxed{\begin{array}{c} u=x+1 \\ du=dx \end{array}} \Rightarrow x=u-1$

invert

$= \boxed{\frac{(x+1)^{16}}{16} - \frac{(x+1)^{15}}{15} + C}$

14. $\int_1^2 \frac{x^2+2}{x^2}\,dx = \int_1^2 1 + 2x^{-2}\,dx = x + \frac{2x^{-1}}{-1}\Big|_1^2 = x - \frac{2}{x}\Big|_1^2$

$= 2-1-(1-2) = 1+1 = \boxed{2}$

15. $\displaystyle\int_2^6 \frac{1}{x^2}\sin\left(\frac{\pi}{x}\right)dx = \frac{-1}{\pi}\int_{\pi/2}^{\pi/6} \sin u\,du = \frac{1}{\pi}\cos u \Big|_{\pi/2}^{\pi/6}$

$$\boxed{\begin{aligned} u &= \pi/x = \pi x^{-1} \\ du &= -\pi/x^2\,dx \\ \frac{-1}{\pi}du &= \frac{1}{x^2}dx \end{aligned}}$$

$$\boxed{\begin{aligned} x=2 &\Rightarrow u=\frac{\pi}{2} \\ x=6 &\Rightarrow u=\pi/6 \end{aligned}}$$

$= \frac{1}{\pi}\left[\cos\frac{\pi}{6} - \cancel{\cos\frac{\pi}{2}}^{\,0}\right]$

$= \frac{1}{\pi}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{3}}{2\pi}}$

16. $\displaystyle\int_1^7 \frac{1}{\sqrt{2x+2}}dx = \frac{1}{2}\int_4^{16}\frac{1}{\sqrt{u}}du = \frac{1}{2}\int_4^{16} u^{-1/2}du = u^{1/2}\Big|_4^{16} = \sqrt{16}-\sqrt{4} = 4-2 = \boxed{2}$

$\frac{\cancel{1}}{\cancel{2}}\frac{u^{1/2}}{\cancel{1/2}}$

$$\boxed{\begin{aligned} u &= 2x+2 \\ du &= 2dx \\ \frac{1}{2}du &= dx \end{aligned}}$$

$$\boxed{\begin{aligned} x=1 &\Rightarrow u=4 \\ x=7 &\Rightarrow u=16 \end{aligned}}$$

17. $\displaystyle\int \frac{x\sin\sqrt{x^2+4}}{\sqrt{x^2+4}}dx = \int \sin u\,du = -\cos u + C = \boxed{-\cos\sqrt{x^2+4}+C}$

$$\boxed{\begin{aligned} u &= \sqrt{x^2+4} \\ du &= \frac{1}{\cancel{2}\sqrt{x^2+4}}(\cancel{2}x)dx = \frac{x}{\sqrt{x^2+4}}dx \end{aligned}}$$

18. $\displaystyle\int_3^5 \frac{x}{(30-x^2)^2}dx = -\frac{1}{2}\int_{21}^5 \frac{1}{u^2}du = -\frac{1}{2}\int_{21}^5 u^{-2}du = -\frac{1}{2}\left(\frac{u^{-1}}{-1}\right)\Big|_{21}^5$

$$\boxed{\begin{aligned} u &= 30-x^2 \\ du &= -2x\,dx \\ -\frac{1}{2}du &= x\,dx \end{aligned}}$$

$$\boxed{\begin{aligned} x=3 &\Rightarrow u=30-9=21 \\ x=5 &\Rightarrow u=30-25=5 \end{aligned}}$$

$= \frac{1}{2u}\Big|_{21}^5$

$= \frac{1}{10} - \frac{1}{42} = \frac{42}{420} - \frac{10}{420}$

$= \frac{32}{420} = \boxed{\frac{8}{105}}$

19. $$\int \frac{\sqrt{7}}{\sqrt{x}(\sqrt{x}+4)^2}\,dx = 2\sqrt{7}\int \frac{1}{u^2}\,du = 2\sqrt{7}\int u^{-2}\,du = 2\sqrt{7}\left(\frac{u^{-1}}{-1}\right)+C$$

$$= \boxed{\frac{-2\sqrt{7}}{\sqrt{x}+4}+C}$$

$u=\sqrt{x}+4$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

20. $$\int x\left(\sqrt{x^2+1}+\frac{1}{\sqrt{x^2+1}}\right)dx = \frac{1}{2}\int \sqrt{u}+\frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int u^{1/2}+u^{-1/2}\,du$$

$$= \frac{1}{2}\left[\frac{2}{3}u^{3/2}+2u^{1/2}\right]+C$$

$$= \boxed{\frac{1}{3}(x^2+1)^{3/2}+\sqrt{x^2+1}+C}$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

21. split

$$\int_0^4 \sqrt{x}+\sqrt{2x+1}\,dx = \int_0^4 \sqrt{x}\,dx + \int_0^4 \sqrt{2x+1}\,dx$$

power rule

u-subst.

$u=2x+1$

$du = 2dx$

$\frac{1}{2}du = dx$

$x=0 \Rightarrow u=1$

$x=4 \Rightarrow u=9$

$u^{1/2}$

$$= \frac{2}{3}x^{3/2}\Big|_0^4 + \frac{1}{2}\int_1^9 \sqrt{u}\,du$$

$$= \frac{2}{3}\left[(4)^{3/2}-0\right] + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\right)u^{3/2}\Big|_1^9$$

$(\sqrt{4})^3$

$$= \frac{2}{3}(8) + \frac{1}{3}\left[9^{3/2}-1^{3/2}\right]$$

$(\sqrt{9})^3$

$$= 16/3 + \frac{1}{3}[27-1] = \frac{16}{3}+\frac{26}{3} = \frac{42}{3} = \boxed{14}$$

22. $\int \sqrt{3}\, x \left(x^2+\pi\right)^{-12/7} dx = \frac{\sqrt{3}}{2}\int u^{-12/7}\, du = \frac{\sqrt{3}}{2}\left(-\frac{7}{5}\right) u^{-5/7} + C$

$$\boxed{\begin{array}{c} u = x^2+\pi \\ du = 2x\, dx \\ \frac{1}{2}\, du = x\, dx \end{array}}$$

$$= \boxed{\frac{-7\sqrt{3}}{10}\left(x^2+\pi\right)^{-5/7} + C}$$

23. $\int_{-1}^{1/2} 1 + x + x^2 + x^3\, dx = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}\Big|_{-1}^{1/2}$

$$= \frac{1}{2} + \frac{(1/2)^2}{2} + \frac{\left(\frac{1}{2}\right)^3}{3} + \frac{\left(\frac{1}{2}\right)^4}{4} - \left[-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4}\right]$$

$$= \cancel{\frac{1}{2}} + \frac{1}{8} + \frac{1}{24} + \frac{1}{64} + 1 - \cancel{\frac{1}{2}} + \frac{1}{3} - \frac{1}{4}$$

$$= \frac{24}{192} + \frac{8}{192} + \frac{3}{192} + \frac{192}{192} + \frac{64}{192} - \frac{48}{192}$$

$$= \frac{291}{192} - \frac{48}{192}$$

$$= \boxed{\frac{243}{192}} = \boxed{\frac{81}{64}}$$

$$\begin{array}{r} 48 \\ 4\overline{)192} \\ 16 \end{array}$$

$24 = 2^3 \cdot 3$

$64 = 2^6$

$$\begin{array}{r} 64 \\ \times 3 \\ \hline 192 \end{array}$$

$$\begin{array}{r} 24 \\ 8\overline{)192} \\ 16 \end{array}$$

$$\begin{array}{r} 24 \\ \times 8 \\ \hline 192 \end{array}$$

$$\begin{array}{r} 2 \\ 24 \\ 8 \\ 3 \\ 192 \\ 64 \\ \hline 291 \\ -48 \\ \hline 243 \end{array}$$

24. $\int_0^{\pi/2} \sec^2\left(\frac{x}{2}\right) dx = 2\int_0^{\pi/4} \sec^2 u\, du = 2\tan u \Big|_0^{\pi/4} = 2\overset{1}{\cancel{\tan \frac{\pi}{4}}} - 2\overset{0}{\cancel{\tan 0}}$

$$= 2 - 0 = \boxed{2}$$

$$\boxed{\begin{array}{c} u = \frac{x}{2} \\ du = \frac{1}{2} dx \\ 2du = dx \end{array}} \quad \boxed{\begin{array}{c} x = 0 \Rightarrow u = 0 \\ x = \frac{\pi}{2} \Rightarrow u = \frac{\pi}{4} \end{array}}$$

25. $\int_{\pi/12}^{\pi/6} \sec(2x)\tan(2x)\,dx = \frac{1}{2}\int_{\pi/6}^{\pi/3} \sec u \tan u\,du = \frac{1}{2}\sec u \Big|_{\pi/6}^{\pi/3}$

$\boxed{\begin{aligned} u &= 2x \\ du &= 2dx \\ \tfrac{1}{2}du &= dx \end{aligned}}$ $\boxed{\begin{aligned} x = \pi/12 &\Rightarrow u = \pi/6 \\ x = \pi/6 &\Rightarrow u = \pi/3 \end{aligned}}$

$= \frac{1}{2}\left[\sec\frac{\pi}{3} - \sec\frac{\pi}{6}\right]$

$= \frac{1}{2}\left[2 - \frac{2}{\sqrt{3}}\right]$

$\cos \pi/3 = 1/2 \qquad \cos \pi/6 = \sqrt{3}/2$

$= \boxed{1 - \frac{1}{\sqrt{3}}} = \boxed{\frac{\sqrt{3}-1}{\sqrt{3}}}$

26. $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = 2\int \sqrt{u}\,du = 2\left(\frac{2}{3}\right)u^{3/2} + C$

$\boxed{\begin{aligned} u &= 1+\sqrt{x} \\ du &= \frac{1}{2\sqrt{x}}dx \\ 2du &= \frac{1}{\sqrt{x}}dx \end{aligned}}$

$= \boxed{\frac{4}{3}\left(1+\sqrt{x}\right)^{3/2} + C}$

27. $\int \sqrt{1+\sqrt{x}}\,dx = 2\int (u-1)\sqrt{u}\,du = 2\int u^{3/2} - u^{1/2}\,du = 2\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + C$

$= \boxed{2\left[\frac{2}{5}\left(1+\sqrt{x}\right)^{5/2} - \frac{2}{3}\left(1+\sqrt{x}\right)^{3/2}\right] + C}$

$\boxed{\begin{aligned} u &= 1+\sqrt{x} \Rightarrow \sqrt{x} = u-1 \\ du &= \frac{1}{2\sqrt{x}}dx \\ 2\sqrt{x}\,du &= dx \quad *\text{invert}* \\ 2(u-1)\,du &= dx \end{aligned}}$

28. $v(t) = t^2 - 11t + 24$

$$\text{Displacement} = \int_0^{10} t^2 - 11t + 24\,dt = \frac{t^3}{3} - \frac{11t^2}{2} + 24t \Big|_0^{10}$$

$$= \frac{1000}{3} - \frac{1100}{2} + 240 - (0 - 0 + 0)$$

$$= \frac{1000}{3} - 550 + 240$$

$$= \frac{1000}{3} - 310$$

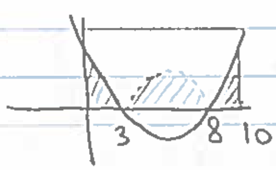
$$= \frac{1000}{3} - \frac{930}{3}$$

$$= \boxed{\frac{70}{3}}$$

$$\text{Total Distance} = \int_0^{10} |t^2 - 11t + 24|\,dt = \int_0^3 |t^2-11t+24|\,dt + \int_3^8 \overset{-(t^2-11t+24)}{|t^2-11t+24|}\,dt + \int_8^{10} |t^2-11t+24|\,d$$

$t^2 - 11t + 24 = (t-3)(t-8)$

$$= \int_0^3 t^2 - 11t + 24\,dt + \int_3^8 -t^2 + 11t - 24\,dt + \int_8^{10} t^2 - 11t + 24\,dt$$



$$= \frac{t^3}{3} - \frac{11t^2}{2} + 24t \Big|_0^3 + \left(-\frac{t^3}{3} + \frac{11t^2}{2} - 24t\right)\Big|_3^8 + \left(\frac{t^3}{3} - \frac{11t^2}{2} + 24t\right)\Big|_8^{10}$$

$$= 9 - \frac{99}{2} + 72 - (0 - 0 + 0) + \left(-\frac{512}{3} + \frac{704}{2} - 192\right) - \left(-9 + \frac{99}{2} - 72\right) + \left(\frac{1000}{3} - \frac{1100}{2} + 240\right) - \left(\frac{512}{3} - \frac{704}{2} + 192\right)$$

$$= 9 - \boxed{\frac{99}{2}} + 72 - \frac{512}{3} + \underbrace{352 - 192}_{160} + 9 - \boxed{\frac{99}{2}} + 72 + \frac{1000}{3} \underbrace{- 550 + 240}_{-310} - \frac{512}{3} + \underbrace{352 - 192}_{160}$$

$$= \frac{-\cancel{24}}{\cancel{3}} + 172 - \frac{198}{2}$$

$$= -8 + 172 - 99 = \boxed{65}$$

Maybe Easier to use Symmetry Available $= 2\int_0^3 t^2 - 11t + 24\,dt + \int_3^8 -(t^2 - 11t + 24)\,dt$

29 Let $W(t)$ = Amount Logs Chucked by WoodChuck at Time t.

Net Change $W(8) - W(0) = \int_0^8 W'(t)\,dt = \int_0^8 2t+1\,dt$

$= t^2 + t \Big|_0^8 = 64 + 8 - (0+0) = \boxed{72}$ logs