Name:_____

**Amherst College**

**DEPARTMENT OF MATHEMATICS**

**Math 106**

**Midterm Exam #1**

**February 16, 2018**

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

| Problem | Score | Possible Points |
|---|---|---|
| 1 | | 22 |
| 2 | | 15 |
| 3 | | 20 |
| 4 | | 15 |
| 5 | | 12 |
| 6 | | 16 |
| Total | | 100 |

**1.** [22 Points] Differentiate each of the following functions. **Do not** simplify your answers.

(a) $f(x) = \cos \pi + \sqrt{\cos \sqrt{x}}$

(b) $f(x) = \cos(\sin x)$

(c) $f(x) = \cos x \cdot \sin x$

(d) $f(x) = \cos^5 \left( \dfrac{7}{x^6} \right)$

(e) $f(x) = \left( \dfrac{\cos(7x)}{\tan(3x)} \right)^{\frac{7}{8}}$

**2.** [15 Points] Derivatives

(a) Let $f(x) = \dfrac{1}{\sin^2 x} + \tan(2x) \cdot \sin(2x)$. Compute $f'\left(\dfrac{\pi}{6}\right)$. Simplify.

(b) Let $f(x) = 4\sin\left(x - \dfrac{\pi}{4}\right) - \cos x - \tan^2 x$. Show that $f'\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$.

**3.** [20 Points] Compute the following **most general antiderivatives.** Do not simplify your final answer.

(a) $\displaystyle\int \frac{5}{6}x + x^{\frac{5}{6}} + \frac{1}{x^{\frac{5}{6}}} + \frac{6}{5} + \frac{5}{6x^6} - \frac{6}{x^5}\ dx$

(b) $\displaystyle\int \sec^2 x - 8\cos x + \sin x + \frac{\sec x \tan x}{7}\ dx$

**3.** (Continued) Compute the following **most general antiderivatives.** Do not simplify your final answer.

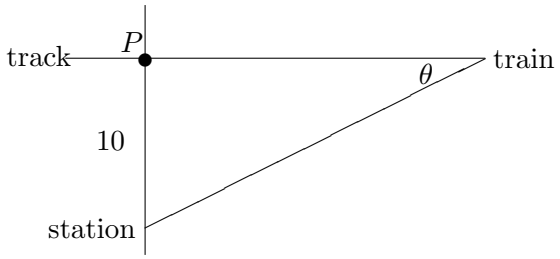
(c) $\displaystyle\int \left(x^2 + \frac{1}{x^2}\right)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)\, dx$

(d) $\displaystyle\int \frac{x^2 + \sqrt{x}}{x^{\frac{3}{7}}}\, dx$

**4.** [15 Points] Consider a point $P$ on a train track. Suppose a train depot station is 10 feet directly south from this point $P$. The train is travelling east at 6 feet per second. Consider the angle as shown in the diagram. How fast is this angle changing when 2 seconds has passed since the train passed point $P$.

• Diagram

The picture at arbitrary time $t$ is:



**5.** [12 Points] Consider a function $f$ such that $f''(x) = \pi \sin x + 2\cos x$ and $f'\left(\frac{\pi}{2}\right) = 0$ and $f(\pi) = 2$. Compute $f(x)$.

**6.** [16 Points] For each of the following use $a(t) = -32$ feet per second squared as acceleration due to gravity on the falling body.

(a) A ball is thrown upwards from the top of a building with an initial *speed* of 32 feet per second. The ball hits the ground below with a *speed* of 64 feet per second. How tall is the building?

**6.** (Continued) For each of the following use $a(t) = -32$ feet per second squared as acceleration due to gravity on the falling body.

(b) A ball is dropped straight down from the top of a building 64 feet tall, with intitial velocity of 0 feet per second. What is the velocity at impact?