• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\dfrac{\pi}{6}\right)$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [20 Points] Differentiate each of the following functions. **Do not** simplify your answers.

(a) $f(x) = \cos\left(\tan^2\left(\dfrac{3}{x^5}\right)\right) + \tan\left(\cos^2\left(\dfrac{x^5}{3}\right)\right)$

(b) $y = \sec^2(7x) \cdot \tan(7x^2)$

(c) $g(t) = \dfrac{\sqrt{t} + \sec(\sqrt{t})}{\sqrt{1 + \sec t}}$

2. [15 Points] Consider the curve given by $\cos(xy^2) + 2 = \sin x + y^3$.

(a) Compute $\dfrac{dy}{dx}$.

(b) Find the equation of the tangent line to this curve at the point $(\pi, 1)$.

3. [20 Points]

(a) Let $f(x) = \sin^3(4x) + \sec(4x) - 8\sin(2x)$.   Compute $f'\left(\dfrac{\pi}{12}\right)$.   Simplify.

(b) Let $f(x) = \cos(2x) + \dfrac{1}{\tan^2 x} + \sin\left(x - \dfrac{\pi}{4}\right)$.   Compute $f'\left(\dfrac{\pi}{4}\right)$.   Simplify.

4. [15 Points] A judge is watching a tennis match. She is sitting on the court, in line with the net, 10 meters from the center of the net. The tennis ball is travelling perpendicular to the net on a path that passes directly over the center of the net. How quickly is the judge rotating her head to keep the ball in sight, when the ball is moving at 2 meters per second and is 3 meters from the net?

• Diagram
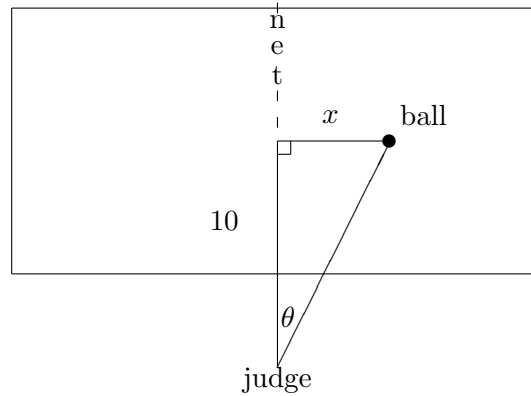
5. [20 Points] Find the general antiderivative for each of the following:

(a) $\dfrac{x^6 + \dfrac{1}{\sqrt{x}}}{x^{\frac{3}{4}}}$

(b) $\left(x^{\frac{2}{3}} + \dfrac{1}{x^2}\right)\left(\sqrt{x} - \dfrac{1}{x^{\frac{3}{5}}}\right)$

6. [10 Points] A nearby cave has an increasing population of bats. An agricultural company has been studying the cave since 2000 because guano (bat poop) makes great fertilizer. The company estimates that the the amount of guano, $G(t)$, has been increasing at a *rate* of $G'(t) = 2 + \dfrac{3}{20}t^2$ tons per year, where t is time in years since 2000. In 2010, ten years later, the company cleaned out the guano and found 80 tons of the stuff. How much guano was in the cave in 2000 at time $t = 0$?