# HW #7

1. (a) $f(x) = \dfrac{\left(\cos x - \frac{1}{\sqrt{x}}\right)^9}{\sec(4x)}$ $\quad x^{-1/2}$

$$f'(x) = \boxed{\dfrac{\sec(4x)\cdot 9\left(\cos x - \frac{1}{\sqrt{x}}\right)^8\left(-\sin x + \frac{1}{2}x^{-3/2}\right) - \left(\cos x - \frac{1}{\sqrt{x}}\right)^9 \sec(4x)\tan(4x)\,4.}{\sec^2(4x)}}$$

(b) $y = \sin^3(x^3) = [\sin(x^3)]^3$

$y' = 3\sin^2(x^3)\cos(x^3)(3x^2) = \boxed{9x^2\sin^2(x^3)\cos(x^3)}$

(c) $f(t) = t^2\sin^5(2t)$

$f'(t) = \boxed{t^2\cdot 5\sin^4(2t)\cos(2t)(2) + \sin^5(2t)(2t)}$

(d) $g(x) = \cos(3x)\sin(4x)$

$g'(x) = \boxed{\cos(3x)\cos(4x)(4) + \sin(4x)(-\sin(3x))(3)}$

(e) $g(x) = \dfrac{\cos(3x)}{\sin(4x)}$

$$g'(x) = \boxed{\dfrac{\sin(4x)(-\sin(3x))(3) - \cos(3x)\cos(4x)4}{\sin^2(4x)}}$$

1. continued

(f) $g(t) = \cos\left(\sin^3\left(\frac{t}{\sqrt{t+1}}\right)\right)$

① cos ② $^3$ ③ sin ④ $\frac{t}{\sqrt{t+1}}$ pieces

$$\boxed{g'(t) = -\sin\left(\sin^3\left(\frac{t}{\sqrt{t+1}}\right)\right)\cdot 3\sin^2\left(\frac{t}{\sqrt{t+1}}\right)\cos\left(\frac{t}{\sqrt{t+1}}\right)\left[\frac{\sqrt{t+1}\,(1) - t\left(\frac{1}{2\sqrt{t+1}}\right)}{\underbrace{t+1}_{(\sqrt{t+1})^2}}\right]}$$

(g) $g(x) = \sqrt{\cos(x^2 - \sin x)} + \sin\sqrt{x^2+\sec x} + \frac{1}{\tan(\sqrt{x}+\cos x)}$

$\frac{1}{\tan(\sqrt{x}+\cos x)} \rightarrow [\tan(\sqrt{x}+\cos x)]^{-1}$

$$\boxed{\begin{aligned} g'(x) = {} & \frac{1}{2\sqrt{\cos(x^2-\sin x)}}\cdot(-\sin(x^2-\sin x))[2x-\cos x] \\ & + \cos\sqrt{x^2+\sec x}\left(\frac{1}{2\sqrt{x^2+\sec x}}\right)(2x+\sec x\tan x) \\ & - [\tan(\sqrt{x}+\cos x)]^{-2}\sec^2(\sqrt{x}+\cos x)\left[\frac{1}{2\sqrt{x}} - \sin x\right] \end{aligned}}$$

$$\sqrt{3}\,[\tan(3x)]^{-2}$$

2. $f(x) = \cos^2(2x) + \sec(4x) + \dfrac{\sqrt{3}}{\tan^2(3x)}$ Compute $f'(\pi/12)$

$$f'(x) = 2\cos(2x)(-\sin(2x))(2) + \sec(4x)\tan(4x)(4) - 2\sqrt{3}\,[\tan(3x)]^{-3}\sec^2(3x)(3)$$

$$= -4\cos(2x)\sin(2x) + 4\sec(4x)\tan(4x) - \frac{6\sqrt{3}\sec^2(3x)}{\tan^3(3x)}$$

$$f'(\pi/12) = -4\underbrace{\cos(\pi/6)}_{\sqrt{3}/2}\underbrace{\sin(\pi/6)}_{1/2} + 4\underbrace{\sec(\pi/3)}_{2}\underbrace{\tan(\pi/3)}_{\sqrt{3}} - \frac{6\sqrt{3}\underbrace{\sec^2(\pi/4)}_{(\sqrt{2})^2}}{\underbrace{\tan^3(\pi/4)}_{1}}$$

$$= -\sqrt{3} + 8\sqrt{3} - 12\sqrt{3}$$

$$= 8\sqrt{3} - 13\sqrt{3} = \boxed{-5\sqrt{3}}$$

$\sec(\pi/4) = \dfrac{1}{\cos \pi/4} = \dfrac{1}{1/\sqrt{2}} = \sqrt{2}$

3. $\sin(xy) = \sec x + \underbrace{\cos(\pi)}_{\text{constant}} - y$ $\quad -1$

Implicit Differentiation

$$\cos(xy)\left[x\frac{dy}{dx} + y\right] = \sec x\tan x + 0 - \frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} + y\cos(xy) = \sec x\tan x - \frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} + \frac{dy}{dx} = \sec x\tan x - y\cos(xy)$$

Factor

$$[x\cos(xy) + 1]\frac{dy}{dx} = \sec x\tan x - y\cos(xy)$$

Solve $\quad \dfrac{dy}{dx} = \boxed{\dfrac{\sec x\tan x - y\cos(xy)}{x\cos(xy) + 1}}$

4 (a) $4\cos x \sin y = 3$ at $(\pi/6, \pi/3)$

$\frac{d}{dx}\left[4\cos x \sin y\right] = \frac{d}{dx}[3]$ Implicit Differentiation

$4[\quad]$

$4\cos x(\cos y)\frac{dy}{dx} + 4\sin y(-\sin x) = 0$

Plug in $x = \pi/6$, $y = \pi/3$

$4\underbrace{\cos \pi/6}_{\sqrt{3}/2}\underbrace{\cos \pi/3}_{1/2}\frac{dy}{dx} + 4\underbrace{\sin \pi/3}_{\sqrt{3}/2}(-\underbrace{\sin \pi/6}_{1/2}) = 0$

$\cancel{4}\left(\frac{\sqrt{3}}{\cancel{2}}\right)\left(\frac{1}{\cancel{2}}\right)\frac{dy}{dx} - \cancel{4}\left(\frac{\sqrt{3}}{\cancel{2}}\right)\left(\frac{1}{\cancel{2}}\right) = 0$.

$\sqrt{3}\frac{dy}{dx} - \sqrt{3} = 0$

Solve $\left.\frac{dy}{dx}\right|_{(\pi/6, \pi/3)} = \frac{\sqrt{3}}{\sqrt{3}} = 1$

Equation $y - \pi/3 = 1(x - \pi/6)$

$y = x - \pi/6 + \pi/3$

$\boxed{y = x + \pi/6}$

(b) $y^3 - xy^2 + \cos(xy) = 2$ at $(0,1)$

$\frac{d}{dx}\left[y^3 - xy^2 + \cos(xy)\right] = \frac{d}{dx}[2]$ Implicit Differentiation

$3y^2\frac{dy}{dx} - \left[x2y\frac{dy}{dx} + y^2(1)\right] - \sin(xy)\left[x\frac{dy}{dx} + y\right] = 0$

Plug in $x = 0$, $y = 1$

$3\frac{dy}{dx} - [0 + 1] - \cancel{\sin 0}[0+1] = 0 \Rightarrow 3\frac{dy}{dx} = 1$

0

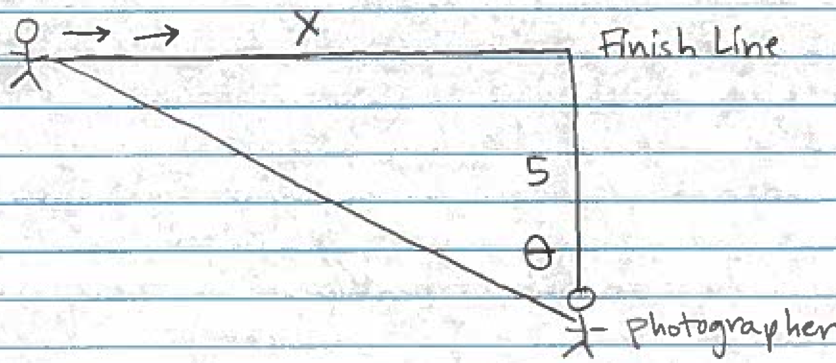
0

Solve $\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{1}{3}$

Equation $y - 1 = \frac{1}{3}(x - 0)$

$\boxed{y = \frac{1}{3}x + 1}$

5. • Diagram




• Variables

Let $x$ = distance between runners and finish line at time $t$.
$\theta$ = angle camera is turned from finish line at time $t$.

Find $\frac{dx}{dt} = ?$ when $x = 12$ and $\frac{d\theta}{dt} = \frac{-3}{5}$ rad/sec.

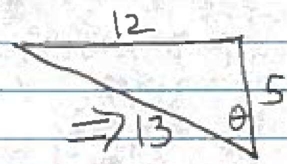
• Equation

$$\tan\theta = \frac{x}{5}$$

• Differentiate

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$$

• Extra Solvable Info




$\Rightarrow \sec\theta = \frac{hyp}{adj} = \frac{13}{5}$

• Substitute

$$\left(\frac{13}{5}\right)^2 \left(\frac{-3}{5}\right) = \frac{1}{5}\frac{dx}{dt}$$

• Solve $\frac{dx}{dt} = \cancel{5}\left(\frac{169}{25}\right)\left(\frac{-3}{\cancel{5}}\right) = \frac{-507}{25}$ yds/sec.

• Answer The runners are moving closer to the finish line at a rate of $\frac{507}{25}$ yards every second.

6. $a(t) = t + \cos t$

$$v(t) = \frac{t^2}{2} + \sin t + C$$

$$v(0) = 0 + \sin 0 + C \overset{\text{set}}{=} 3 \Rightarrow C = 3$$

$$v(t) = t^2/2 + \sin t + 3$$

$$s(t) = \frac{t^3}{6} - \cos t + 3t + C^*$$

$$s(0) = 0 - \cancel{\cos 0}^{\,1} + 0 + C^* \overset{\text{set}}{=} -2$$

$$-1 + C^* = -2 \Rightarrow \text{Solve } C^* = -1$$

Finally $s(t) = \boxed{t^3/6 - \cos t + 3t - 1}$ feet.

7. $f'(t) = t^2(1+t) + 2\sin t \quad f(0) = 3.$

$$f'(t) = t^2 + t^3 + 2\sin t$$

$$f(t) = \frac{t^3}{3} + \frac{t^4}{4} - 2\cos t + C$$

$$f(0) = 0 + 0 - 2\cancel{\cos 0}^{\,1} + C \overset{\text{set}}{=} 3$$

$$-2 + C = 3 \Rightarrow C = 5$$

Finally $\boxed{f(t) = t^3/3 + t^4/4 - 2\cos t + 5}$

$\frac{1}{2} - \frac{3}{7} = \frac{7}{14} - \frac{6}{14} = \frac{1}{14}$

8. $\dfrac{x^2+\sqrt{x}}{x^{3/7}} = \dfrac{x^2}{x^{3/7}} + \dfrac{\sqrt{x}}{x^{3/7}} = x^{11/7} + x^{1/14}$

$14/7$ $1/2$ $7/14$

Antidifferentiate $\dfrac{x^{18/7}}{\left(\frac{18}{7}\right)} + \dfrac{x^{15/14}}{\left(\frac{15}{14}\right)} + C = \boxed{\frac{7}{18}x^{18/7} + \frac{14}{15}x^{15/14} + C}$

9. $\left(x^2+\frac{1}{x^2}\right)\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right) = x^2\sqrt{x} + \dfrac{\sqrt{x}}{x^2} + \dfrac{x^2}{\sqrt{x}} + \dfrac{1}{x^2\sqrt{x}}$

$= x^{5/2} + x^{-3/2} + x^{3/2} + x^{-5/2}$

Antidifferentiate

$\dfrac{x^{7/2}}{c\left(\frac{7}{2}\right)} + \dfrac{x^{-1/2}}{c\left(\frac{-1}{2}\right)} + \dfrac{x^{5/2}}{c\left(\frac{5}{2}\right)} + \dfrac{x^{-3/2}}{c\left(\frac{-3}{2}\right)} + C$

$$\boxed{\frac{2}{7}x^{7/2} - 2x^{-1/2} + \frac{2}{5}x^{5/2} - \frac{2}{3}x^{-3/2} + C}$$