**Math 106 Final Examination May 13, 2014**

**1.** [30 Points] Compute each of the following derivatives.

(a) $f''(1)$, where $f(x) = \dfrac{\ln x}{x}$

(b) $\dfrac{d}{dx} \ln\left(\dfrac{e^{3-x}\ \sqrt{\sec x + \sqrt{x}}}{e^{\ln x}\ (4-x^4)^4}\right)$ Do not simplify the final answer here.

(c) $g'(x)$, where $g(x) = (\sin x)^{\frac{1}{x}}$

(d) $\dfrac{dy}{dx}$, if $\sec^2 y - \cos x = e^{x^2 y} + (\ln(e+5))x$

(e) $y'$, where $y = \dfrac{1}{\sin\sqrt{e^x + e^7}} + \dfrac{1}{e^{\sqrt{x^2+7\sin x}}} + \dfrac{1}{\sqrt{7\sin x + e^{x^2}}}$ Do not simplify.

**2.** [20 Points] Compute each of the following derivatives.

(a) $f'\left(\dfrac{\pi}{6}\right)$ where $f(x) = \sec(2x) + \cos^2(2x) + \dfrac{2}{\sin x}$

(b) $f'\left(\dfrac{\pi}{3}\right)$ where $f(x) = \cos(3x - \pi) + \sec\left(\dfrac{\pi}{3}\right) + 3\sin\left(x - \dfrac{\pi}{3}\right)$

(c) $f'\left(\dfrac{\pi}{4}\right)$ where $f(x) = \cos(2x) - \sin(2x) + 2\tan x$

**3.** [35 Points] Compute each of the following integrals.

(a) $\displaystyle\int \frac{(1-\sqrt{x})(x^{\frac{1}{3}} - x)}{x^2}\, dx$ (b) $\displaystyle\int x\ (3x-1)^{2014}\ dx$ (c) $\displaystyle\int_1^2 \frac{x}{x^2-9}\, dx$ (d) $\displaystyle\int \frac{1}{x^2 e^{\frac{1}{x}}}\, dx$

(e) $\displaystyle\int_{e^3}^{e^8} \frac{5}{x\sqrt{1+\ln x}}\, dx$ (f) $\displaystyle\int_2^6 \frac{\sin\left(\frac{\pi}{x}\right)\cos\left(\frac{\pi}{x}\right)}{x^2}\, dx$ (g) $\displaystyle\int_{\ln 2}^{\ln 3} \frac{1}{e^{2x}(1-e^{-2x})^2}\, dx$

**4.** [10 Points] Find the equation of the tangent line to the curve

$y = \sin(\ln(1+x)) - \ln(1+\sin x) - \dfrac{e}{1+\ln(1+x)} + \sin(e^{3x} - 1)$ at the point where $x = 0$.

**5.** [20 Points] Consider the function given by

$f(x) = e^x + \dfrac{1}{e^x} + x^e + \dfrac{1}{x^e} + e^e + \dfrac{1}{e^e} + \dfrac{x}{e} + \dfrac{e}{x} + ex + \dfrac{1}{ex}$

(a) Compute the **derivative**, $f'(x)$. (b) Compute the **antiderivative**, $\displaystyle\int f(x)\ dx$.

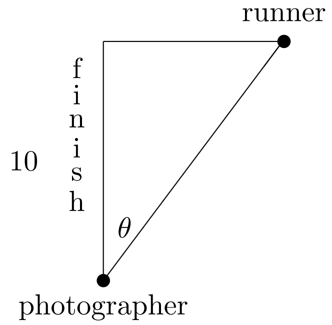
**6.** [15 Points] Compute $\displaystyle\int_1^4 4 - x - x^2\ dx$ using each of the following **two** different methods:

($a$) Fundamental Theorem of Calculus,
($b$) The limit definition of the definite integral.

**Recall** $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\displaystyle\sum_{i=1}^{n} 1 = n$

**7.** [10 Points] A photographer is televising a race from a position that is 10 feet from the track in line with the finish line. When the lead runner is 8 feet from the finish line, he is jogging at a rate of 4 feet per second. At this moment, how fast is the photographer rotating to keep the runner in direct view of the camera?

• Diagram




**8.** [15 Points] Let $\boldsymbol{f(x) = (x-2)^2 e^x}$.

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve. Take my word that $\lim_{x\to\infty} f(x) = +\infty$ and $\lim_{x\to-\infty} f(x) = 0$. Also take my word that $f'(x) = x(x-2)e^x$ and $f''(x) = (x^2-2)e^x$.

**9.** [20 Points] (a) Consider the region bounded by $y = e^x + 1$, $y = 4$, and $x = 0$. Sketch the bounded region. (b) Compute the area of the bounded region in (a).

(c) Compute the volume of the three-dimensional solid obtained by rotating the region in (a) about the $\boldsymbol{x}$**-axis**. Sketch the solid, along with one of the approximating washers.

(d) Consider a **different** region bounded by $y = \sin x$, $y = 1$, $x = 0$ and $x = \frac{\pi}{2}$. Sketch the bounded region.

(e) **Set-Up** but **DO NOT EVALUATE** the integral to compute the volume of the three-dimensional solid obtained by rotating the region in (d) about the horizontal line $\boldsymbol{y = -1}$. Sketch the solid, along with one of the approximating washers.

(f) Consider a **different** region bounded by $y = \ln x$, $y = 0$, $y = 3$ and $x = 0$. Sketch the bounded region.

(g) **Set-Up** but **DO NOT EVALUATE** the integral to compute the volume of the three-dimensional solid obtained by rotating the region in (f) about the $\boldsymbol{y}$**-axis**. Sketch the solid, along with one of the approximating disks.

**10.** [15 Points] Consider an object moving on a number line such that its velocity at time $t$ seconds is given by $\boldsymbol{v(t) = t^2 - 2t}$ feet per second. Also assume that the position of the object at one second is three feet.

(a) Compute the acceleration function $a(t)$ and the position function $s(t)$.

(b) Sketch $v(t)$. Next sketch $|v(t)|$. (c) Compute the **total distance** travelled for $0 \le t \le 3$.

**11.** [10 Points] A population of bacteria was growing exponentially. Initially there were 2 cells. After 1 hour there were 8 cells. How many cells were there after 4 hours? When were there 128 cells?