

### Extra Riemann Sum, Definite Integral Example

Evaluate  $\int_1^5 5 - 2x - x^2 dx$  using Riemann Sums and the limit definition of the definite integral.

Here  $f(x) = 5 - 2x - x^2$ ,  $a = 1$ ,  $b = 5$ ,  $\Delta x = \frac{5-1}{n} = \frac{b-a}{n} = \frac{4}{n}$   
and  $x_i = a + x_i = 1 + i \left(\frac{4}{n}\right) = 1 + \frac{4i}{n}$ .

$$\begin{aligned}\int_1^5 5 - 2x - x^2 dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(1 + \frac{4i}{n}\right) \frac{4}{n} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n 5 - 2\left(1 + \frac{4i}{n}\right) - \left(1 + \frac{4i}{n}\right)^2 \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left(5 - 2 - \frac{8i}{n} - 1 - \frac{8i}{n} - \frac{16i^2}{n^2}\right) \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left(2 - \frac{16i}{n} - \frac{16i^2}{n^2}\right) \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n 2 - \frac{4}{n} \sum_{i=1}^n \frac{16i}{n} - \frac{4}{n} \sum_{i=1}^n \frac{16i^2}{n^2} \\ &= \lim_{n \rightarrow \infty} \frac{8}{n} \sum_{i=1}^n 1 - \frac{64}{n^2} \sum_{i=1}^n i - \frac{64}{n^3} \sum_{i=1}^n i^2 \\ &= \lim_{n \rightarrow \infty} \frac{8}{n}(n) - \frac{64}{n^2} \left(\frac{n(n+1)}{2}\right) - \frac{64}{n^3} \left(\frac{n(n+1)(2n+1)}{6}\right) \\ &= \lim_{n \rightarrow \infty} 8 - \frac{64}{2} \left(\frac{n}{n}\right) \left(\frac{n+1}{n}\right) - \frac{64}{6} \left(\frac{n}{n}\right) \left(\frac{n+1}{n}\right) \left(\frac{2n+1}{n}\right) \\ &= \lim_{n \rightarrow \infty} 8 - 32(1) \left(1 + \frac{1}{n}\right) - \frac{32}{3}(1) \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \\ &= 8 - 32(1)(1) - \frac{32}{3}(1)(1)(2) = 8 - 32 - \frac{64}{3} = -24 - \frac{64}{3} \\ &= -\frac{72}{3} - \frac{64}{3} = \boxed{-\frac{136}{3}}\end{aligned}$$