

Worksheet 7, Tuesday, March 25th, 2025

Limit Definition of the Definite Integral

1. Show that $\int_1^5 7 - x - x^2 dx = \boxed{-\frac{76}{3}}$ using two different methods:

- (a) Fundamental Theorem of Calculus (b) Limit Definition of the Definite Integral.

Differentiation

2. Compute $g'(x)$ where $g(x) = \int_x^2 \frac{\cos t}{5 + \cos t} dt$

Integration Evaluate each of the following integrals:

3. $\int_0^4 \frac{1-x}{\sqrt{x}} dx$ 4. $\int x^4 (2 - 3x^5)^6 dx$ 5. $\int_9^{64} \frac{5}{\sqrt{x}\sqrt{1+\sqrt{x}}} dx$

6. $\int_0^{\frac{\pi}{6}} \frac{\cos x}{(1 + 6 \sin x)^2} dx$ 7. $\int \frac{5}{x^2 \left(5 + \frac{3}{x}\right)^{\frac{3}{5}}} dx$ 8. $\int x(x-2)^{\frac{3}{4}} dx$

Initial Valued Problem

9. Find the function $f(x)$ that satisfies $f'(x) = \frac{\sec^2 x}{\sqrt{3 + \tan x}}$ and $f\left(\frac{\pi}{4}\right) = -5$

Displacement–Total Distance

10. Consider an object moving on the number line such that its Velocity at time t is $v(t) = \sin t$ feet per second. Also assume that $s(0) = 2$ feet, where as usual $s(t)$ is the Position of the object at time t .

- (a) Compute the Acceleration function $a(t)$ and the Position function $s(t)$.
- (b) Draw the graph of $v(t)$ for $0 \leq t \leq 2\pi$, and explain why the object is *not* always moving to the right.
- (c) Draw the graph of $|v(t)| = |\sin t|$ for $0 \leq t \leq 2\pi$.
- (d) Set-Up, but do not compute, the Definite Integral that represents each of the **Displacement** and **Total Distance** traveled for $0 \leq t \leq 2\pi$. Think about how to compute these definite integrals.

Turn in your own solutions into Gradescope before 11:59 pm today, Tuesday March 25

Finish all problems through number 9