

Worksheet 5, Tuesday, March 4, 2025

NOTE: Unless instructions specify to use the Limit Definition of the Definite Integral, you may use the *Quicker* Fundamental Theorem of Calculus, Part II.

1. Use the **Limit Definition of the Derivative** to Compute $\int_{-1}^2 2 - 3x - x^2 dx = \boxed{-\frac{3}{2}}$

Compute each of the following Definite Integrals. Simplify.

2. $\int_0^{\frac{\pi}{3}} \sec^2 \theta d\theta$

3. $\int_{-\pi}^{\frac{\pi}{3}} 7 \cos x dx$

4. $\int_{-2}^{-1} x - \frac{5}{x^3} dx$

5. $\int_0^{\frac{\pi}{6}} (\tan x + \sec x) \sec x dx$

6. $\int_1^2 \left(x - \frac{1}{x}\right)^2 dx$

7. $\int_1^4 \frac{\sqrt{x} - x^2}{x} dx$

8. Show that $\int_{-\pi}^{\pi} \sin x dx = \boxed{0}$. Explain why that makes sense?

9. Compute $\int_2^5 x^2 dx$ using each of the following two methods:

(a) The Fundamental Theorem of Calculus.

(b) The *Limit Definition* of the Definite Integral

Formulas

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

Turn in your own solutions into Gradescope before 11:59 pm today, Tuesday March 4

Finish all problems through number 7

Extra Example of Limit Definition for the Definite Integral

Evaluate $\int_1^5 5 - 2x - x^2 dx$ using the Limit Definition of the Definite Integral.

Here $f(x) = 5 - 2x - x^2$, $a = 1$, $b = 5$, $\Delta x = \frac{5-1}{n} = \frac{b-a}{n} = \frac{4}{n}$

and $x_i = a + i\Delta x = 1 + i\left(\frac{4}{n}\right) = 1 + \frac{4i}{n}$.

$$\begin{aligned}
 \int_1^5 5 - 2x - x^2 dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(1 + \frac{4i}{n}\right) \frac{4}{n} \\
 &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n 5 - 2\left(1 + \frac{4i}{n}\right) - \left(1 + \frac{4i}{n}\right)^2 \quad \text{FOIL algebra} \\
 &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left(5 - 2 - \frac{8i}{n} - 1 - \frac{8i}{n} - \frac{16i^2}{n^2}\right) \quad \text{Simplify} \\
 &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left(2 - \frac{16i}{n} - \frac{16i^2}{n^2}\right) \quad \text{Split Sums} \\
 &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n 2 - \frac{4}{n} \sum_{i=1}^n \frac{16i}{n} - \frac{4}{n} \sum_{i=1}^n \frac{16i^2}{n^2} \quad \text{Factor out non-}i \text{ terms} \\
 &= \lim_{n \rightarrow \infty} \frac{8}{n} \sum_{i=1}^n 1 - \frac{64}{n^2} \sum_{i=1}^n i - \frac{64}{n^3} \sum_{i=1}^n i^2 \quad \text{Use } i \text{ Sum formulas} \\
 &= \lim_{n \rightarrow \infty} \frac{8}{n}(n) - \frac{64}{n^2} \left(\frac{n(n+1)}{2}\right) - \frac{64}{n^3} \left(\frac{n(n+1)(2n+1)}{6}\right) \quad \text{Repartner} \\
 &= \lim_{n \rightarrow \infty} 8 - \frac{64}{2} \left(\frac{n}{n}\right) \left(\frac{n+1}{n}\right) - \frac{64}{6} \left(\frac{n}{n}\right) \left(\frac{n+1}{n}\right) \left(\frac{2n+1}{n}\right) \quad \text{Split-Split} \\
 &= \lim_{n \rightarrow \infty} 8 - 32(1) \left(1 + \frac{1}{n}\right) - \frac{32}{3}(1) \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \quad \text{Finish Limits!} \\
 &= 8 - 32(1) - \frac{32}{3}(1)(2) = -24 - \frac{64}{3} = -\frac{72}{3} - \frac{64}{3} = \boxed{-\frac{136}{3}}
 \end{aligned}$$