Math 106, Spring 2024

Worksheet 5, Tuesday, March 5, 2024

NOTE: Unless instructions specify to use the Limit Definition of the Definite Integral, you may use the *Quicker* Fundamental Theorem of Calculus, Part II.

1. Use the **Limit Definition of the Derivative** to Compute $\int_{-1}^{2} 2 - 3x - x^2 \, dx = \boxed{-\frac{3}{2}}$

Compute each of the following Definite Integrals. Simplify.

2. $\int_{0}^{\frac{\pi}{3}} \sec^2\theta \, d\theta$

3. $\int_{-\pi}^{\frac{\pi}{3}} 7\cos x \, dx$

4. $\int_{-2}^{-1} x - \frac{5}{x^3} \, dx$

5. $\int_{0}^{\frac{\pi}{6}} (\tan x + \sec x)\sec x \, dx$

6. $\int_{1}^{2} \left(x - \frac{1}{x}\right)^2 \, dx$

7. $\int_{1}^{4} \frac{\sqrt{x} - x^2}{x} \, dx$

8. Show that $\int_{-\pi}^{\pi} \sin x \, dx = \boxed{0}$. Explain why that makes sense?

9. Compute $\int_{2}^{5} x^2 \, dx$ using each of the following two methods:
(a) The Fundamental Theorem of Calculus.
(b) The *Limit Definition* of the Definite Integral

Formulas

$$\boxed{\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x}$$

where $\boxed{\Delta x = \frac{b-a}{n}}$ and $\boxed{x_i = a + i\Delta x}$

$$\boxed{\sum_{i=1}^{n} 1 = n} \qquad \boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}} \qquad \boxed{\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}}$$

Turn in your own solutions into Gradescope before 11:59 pm today, Tuesday March 5

Finish all problems through number 7

Extra Example of Limit Definition for the Definite Integral

Evaluate $\int_1^5 5 - 2x - x^2 \, dx$ using the Limit Definition of the Definite Integral.

Here $f(x) = 5 - 2x - x^2$, $\quad a = 1$, $\quad b = 5$, $\quad \Delta x = \frac{5-1}{n} = \frac{b-a}{n} = \frac{4}{n}$

and $x_i = a + i\Delta x = 1 + i\left(\frac{4}{n}\right) = 1 + \frac{4i}{n}$.

$$\int_1^5 5 - 2x - x^2 \, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + \frac{4i}{n}\right)\frac{4}{n}$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} 5 - 2\left(1 + \frac{4i}{n}\right) - \left(1 + \frac{4i}{n}\right)^2 \quad \text{FOIL algebra}$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(5 - 2 - \frac{8i}{n} - 1 - \frac{8i}{n} - \frac{16i^2}{n^2}\right) \quad \text{Simplify}$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(2 - \frac{16i}{n} - \frac{16i^2}{n^2}\right) \quad \text{Split Sums}$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} 2 - \frac{4}{n} \sum_{i=1}^{n} \frac{16i}{n} - \frac{4}{n} \sum_{i=1}^{n} \frac{16i^2}{n^2} \quad \text{Factor out non-}i\text{ terms}$$

$$= \lim_{n\to\infty} \frac{8}{n} \sum_{i=1}^{n} 1 - \frac{64}{n^2} \sum_{i=1}^{n} i - \frac{64}{n^3} \sum_{i=1}^{n} i^2 \quad \text{Use } i \text{ Sum formulas}$$

$$= \lim_{n\to\infty} \frac{8}{n}(n) - \frac{64}{n^2}\left(\frac{n(n+1)}{2}\right) - \frac{64}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) \quad \text{Repartner}$$

$$= \lim_{n\to\infty} 8 - \frac{64}{2}\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{n+1}{n}\right) - \frac{64}{6}\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) \quad \text{Split-Split}$$

$$= \lim_{n\to\infty} 8 - 32\,(1)\left(1 + \cancelto{0}{\frac{1}{n}}\right) - \frac{32}{3}\,(1)\left(1 + \cancelto{0}{\frac{1}{n}}\right)\left(2 + \cancelto{0}{\frac{1}{n}}\right) \quad \text{Finish Limits!}$$

$$= 8 - 32(1) - \frac{32}{3}(1)(2) = -24 - \frac{64}{3} = -\frac{72}{3} - \frac{64}{3} = \boxed{-\frac{136}{3}}$$