Math 106, Spring 2025

Worksheet 2, Tuesday, February 11th, 2025
Antiderivatives

Definition: Take F, f functions defined on an interval I and suppose that $F'(x) = f(x)$ on I. Then

- $F(x)$ is called **AN** Antiderivative of $f(x)$
- $F(x) + C$ is called **the Most General** Antiderivative of $f(x)$, where C is any constant Real Number.

We will use the notation $\displaystyle\int f(x)\, dx$ to denote the Most General Antiderivative. The symbols $\displaystyle\int$ and dx are partnered-together *instructions* to compute the Most General Antiderivative.

For example: $\displaystyle\int x^7\, dx = \frac{x^8}{8} + C$ where $+C$ represent all possible constants.

This means that $\displaystyle\frac{d}{dx}\left(\frac{x^8}{8} + C\right) = x^7$ which looks correct to us.

Note that $\frac{x^8}{8} + 3$ is **an** antiderivative of x^7. So is $\frac{x^8}{8} + 2022$ as well as $\frac{x^8}{8} - 5$ and $\frac{x^8}{8} + \sqrt{3}$.

Hint: if you ever want to know whether you found the correct antiderivative, take the derivative of your answer and check that you return to the original function.

1. Write a *general power rule* for $\displaystyle\int x^n\, dx$ where n is any real number with $n \neq -1$.
(We will learn the $n = -1$ case at the very end of this semester.)

Compute each of the following Most General Antiderivatives

2. $\displaystyle\int \sqrt{x}\, dx$ 3. $\displaystyle\int \frac{1}{x^9}\, dx$ 4. $\displaystyle\int \frac{1}{\sqrt{x}}\, dx$ 5. $\displaystyle\int \frac{1}{x^{\frac{3}{7}}}\, dx$

6. $\displaystyle\int \cos x\, dx$ 7. $\displaystyle\int \sin x\, dx$ 8. $\displaystyle\int \sec^2 x\, dx$ 9. $\displaystyle\int \sec x \tan x\, dx$

For $\#10 - 11$, consider $f(x) = \frac{3}{4}x + x^{\frac{3}{4}} - \frac{1}{x^{\frac{3}{4}}} + \frac{4}{3} + \frac{1}{x^{\frac{4}{3}}} + \frac{3}{4x^4} - \frac{4}{x^3}$ Hint: prep $\frac{1}{x^a} = x^{-a}$

10. Compute $f'(x) = \displaystyle\frac{d}{dx}\left(\frac{3}{4}x + x^{\frac{3}{4}} - \frac{1}{x^{\frac{3}{4}}} + \frac{4}{3} + \frac{1}{x^{\frac{4}{3}}} + \frac{3}{4x^4} - \frac{4}{x^3}\right)$

11. Compute $\displaystyle\int f(x)\, dx = \int \frac{3}{4}x + x^{\frac{3}{4}} - \frac{1}{x^{\frac{3}{4}}} + \frac{4}{3} + \frac{1}{x^{\frac{4}{3}}} + \frac{3}{4x^4} - \frac{4}{x^3}\, dx$

Find the Most General Antiderivative of the following functions. You may need to use some Algebra to *prep* the function pieces for each *Power Rule* and/or *Sum or Difference* Rule(s).

12. $\displaystyle\int x^3(1+x^2)\ dx$

13. $\displaystyle\int \frac{x+\sqrt{x}+7}{x^3}\ dx$ Hint: Split $\displaystyle\frac{a+b}{c} = \frac{a}{b} + \frac{c}{d}$

14. $\displaystyle\int x^2 + x(1+x)^2\ dx$

15. $\displaystyle\int -3\cos x - \sec^2 x - 7\sec x \tan x - \sin x\ dx$

16. Consider the curve $y = 2x + \sin x$. Explain why the tangent lines of this curve are never horizontal.

Find the indicated functions $f(x)$ that also satisfies the given conditions:

17. $f'(x) = 2 + \sin x$ and $f\left(\frac{\pi}{2}\right) = 3$.

18. $f'(x) = x^2 + 1$ and $f(1) = 3$.

19. $f'(x) = x(2+\sqrt{x})$ and $f(4) = 30$.

20. $f''(x) = \dfrac{1}{\sqrt{x}} + 3x^2$ and $f'(1) = 2$, $f(1) = 0$.

21. Can you use a *guess and check* approach to compute the function $f(x)$ where $f'(x) = \sin(3x)$? Check your answer. How? Think about *reversing the Chain Rule here...*

22. CHALLENGE: Can you use a *guess and check* approach to compute the function $f(x)$ where $f'(x) = \dfrac{\sec x \tan x}{\sqrt{\sec x + 8}}$ and $f(0) = 7$? Check your answer.

Turn in your own solutions into Gradescope before 11:59 pm today, Tuesday Feb 11

Finish at least through number 18