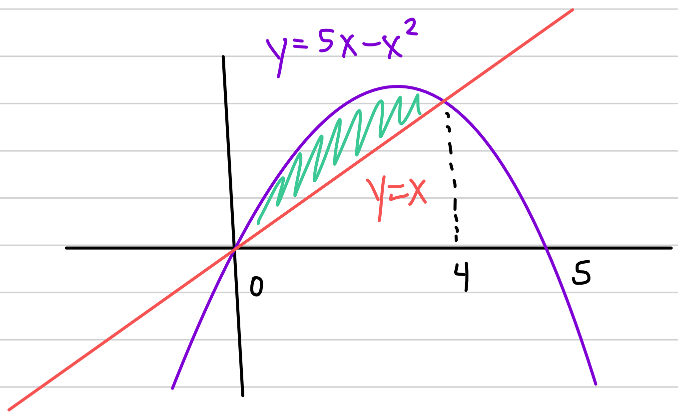
Worksheet 8 Answer Key

1. $y = 5x - x^2$ set $= x(5-x) = 0$ $y = x$

Zeroes: $x = 0$, $x = 5$

Intersect?

$$5x - x^2 = x$$
$$x^2 - 5x + x = 0$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0 \quad x = 4$$



$$\text{Area} = \int_0^4 \text{Top} - \text{Bottom}\, dx = \int_0^4 (5x - x^2) - x\, dx = \int_0^4 4x - x^2\, dx$$

$$= \frac{\cancel{4}^{2} x^2}{\cancel{2}} - \frac{x^3}{3} \Big|_0^4 = 2(4)^2 - \frac{4^3}{3} - (0 - 0)$$

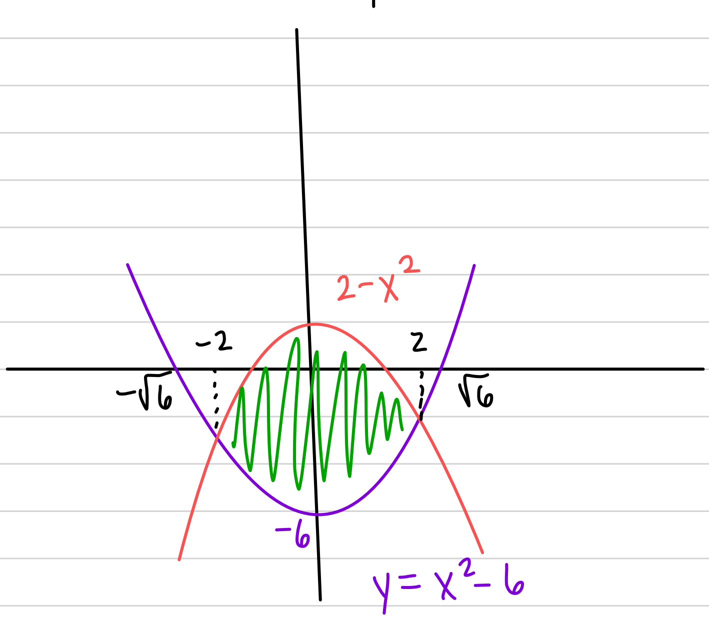
$$= 32 - \frac{64}{3} = \frac{96}{3} - \frac{64}{3} = \frac{32}{3}$$

$+$

2. $= 2 - x^2$ $y = x^2 - 6$

Intersect?

$$2 - x^2 = x^2 - 6$$
$$8 = 2x^2$$
$$4 = x^2$$
$\hookrightarrow x = \pm 2$ Symmetry here



$$\text{Area} = \int_{-2}^{2} \text{Top} - \text{Bottom}\, dx = \int_{-2}^{2} 2 - x^2 - (x^2 - 6)\, dx = \int_{-2}^{2} 2 - x^2 - x^2 + 6\, dx$$

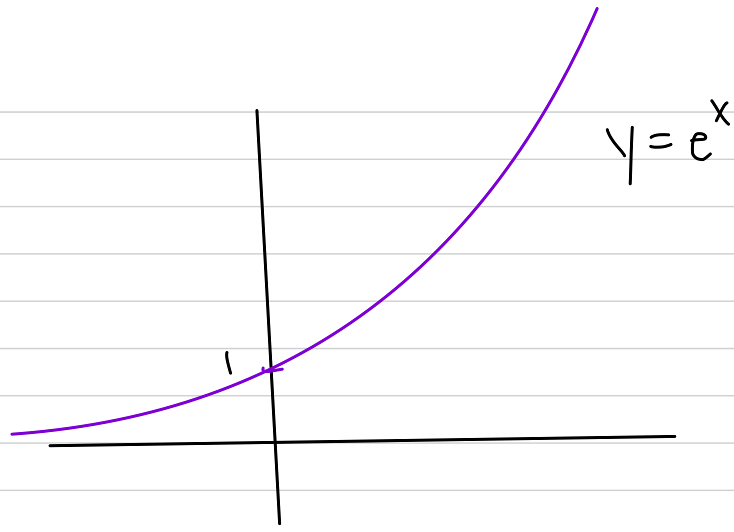
$$= \int_{-2}^{2} 8 - 2x^2\, dx = 8x - \frac{2x^3}{3} \Big|_{-2}^{2}$$

$$= 16 - \frac{2}{3} \cdot 8 - \left(-16 - \frac{2}{3}(-8)\right)$$

$$= 16 - \frac{16}{3} + 16 - \frac{16}{3} = 32 - \frac{32}{3} = \frac{96}{3} - \frac{32}{3} = \frac{64}{3}$$

$+$

3.



Domain: $\mathbb{R}$ or $(-\infty,\infty)$

Range: $(0,\infty)$

4. $\lim_{x\to\infty} e^x = \infty$ $\lim_{x\to-\infty} e^x = 0$

Blows up! Shrinks to 0

5. $f(x)=e^x$ $f'(x)=e^x$

6. $f(x)=\frac{1}{e^x} \overset{\text{prep}}{=} e^{-x}$ $f'(x)=e^{-x}\cdot(-1) = -e^{-x} = \frac{-1}{e^x}$

7. $f(x)=e^{3x}$ $f'(x)=e^{3x}\cdot 3 = 3e^{3x}$

8. $f(x)=\frac{1}{e^{7x}} \overset{\text{prep}}{=} e^{-7x}$ $f'(x)=e^{-7x}(-7) = -7e^{-7x} = \frac{-7}{e^{7x}}$

9. $f(x)=e^{\sin x}$ $f'(x)=\boxed{e^{\sin x}\cdot \cos x}$

10. $f(x)=\sin(e^x)$ $f'(x)=\cos(e^x)\cdot e^x$

11. $f(x)=e^{\sqrt{x}}$ $f'(x)=\boxed{e^{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}}$

12. $f(x)=\sqrt{e^x}$ $f'(x)=\boxed{\frac{1}{2\sqrt{e^x}}\cdot e^x} \overset{\text{or}}{=} \frac{\sqrt{e^x}}{2}$

or prep $f(x)=\sqrt{e^x}=(e^x)^{1/2}=e^{x/2} \Rightarrow f'(x)=e^{x/2}\cdot\frac{1}{2}=\frac{1}{2}\sqrt{e^x}$ Match!

13. $f(x)=e^{(e^x)}$ $f'(x)=\boxed{e^{(e^x)}\ e^x}$

14. $f(x)=e$ constant $f'(x)=0$

15. $f(x)=\frac{e}{x}=ex^{-1}$ $f'(x)=e(-x^{-2})=-\frac{e}{x^2}$

16. $f(x)=\frac{x}{e}=\frac{1}{e}\cdot x$ $f'(x)=\frac{1}{e}$

17. $f(x) = e^5$ constant $f'(x) = 0$

18. $f(x) = ex$ $f'(x) = e$

19. $f(x) = \frac{1}{ex} = \frac{1}{e} \cdot x^{-1}$ $f'(x) = \frac{1}{e}\left(-x^{-2}\right) = -\frac{1}{ex^2}$

20. $f(x) = x^e$ $f'(x) = e\,x^{e-1}$ power rule, e = constant

21. $f(x) = \frac{1}{x^e} = x^{-e}$ $f'(x) = -e\,x^{-e-1}$ NOT exponential rule

22. $f(x) = \frac{e^{-2x}}{1+e^x}$ $f'(x) = \frac{(1+e^x)\cdot e^{-2x}(-2) - e^{-2x}\cdot e^x}{(1+e^x)^2} = \frac{-2e^{-2x} - 2e^{-x} - e^{-x}}{(1+e^x)^2}$

$$= \boxed{\frac{-2e^{-2x} - 3e^{-x}}{(1+e^x)^2}}$$

23. $f(x) = \left(e^{2x} - e^{-3x}\right)^7$ $f'(x) = 7\left(e^{2x} - e^{-3x}\right)^6 \cdot \left(e^{2x}\cdot 2 - e^{-3x}\cdot(-3)\right)$

$$= \boxed{7\left(e^{2x} - e^{-3x}\right)^6 \left(2e^{2x} + 3e^{-3x}\right)}$$

24. $e^{xy} = 2 + \sin x$ Implicit Differentiation

$$\frac{d}{dx}\left(e^{xy}\right) = \frac{d}{dx}\left(2 + \sin x\right)$$

$$e^{xy}\cdot\left(x\cdot\frac{dy}{dx} + y(1)\right) = \cos x$$

$$xe^{xy}\frac{dy}{dx} + ye^{xy} = \cos x$$

$$xe^{xy}\frac{dy}{dx} = \cos x - ye^{xy}$$

Solve: $\frac{dy}{dx} = \boxed{\frac{\cos x - ye^{xy}}{xe^{xy}}}$

25. $\int e^x\sqrt{1-e^x}\,dx = -\int\sqrt{u}\,du = -\int u^{1/2}\,du = -\frac{u^{3/2}}{3/2} + C = \boxed{-\frac{2}{3}\left(1-e^x\right)^{3/2} + C}$

$$\boxed{\begin{aligned} u &= 1-e^x \\ du &= -e^x dx \\ -du &= e^x dx \end{aligned}}$$