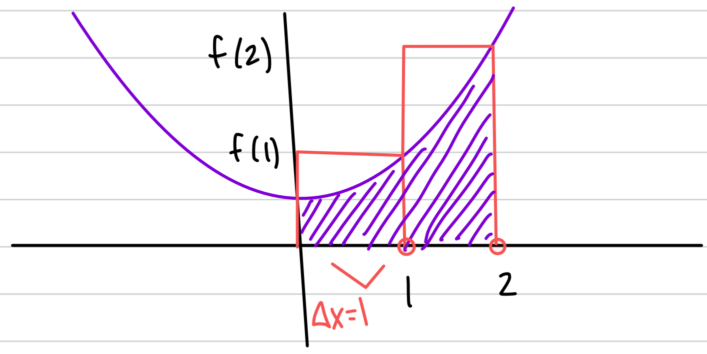
Worksheet 4 Answer Key

1. $f(x) = x^2 + 1$

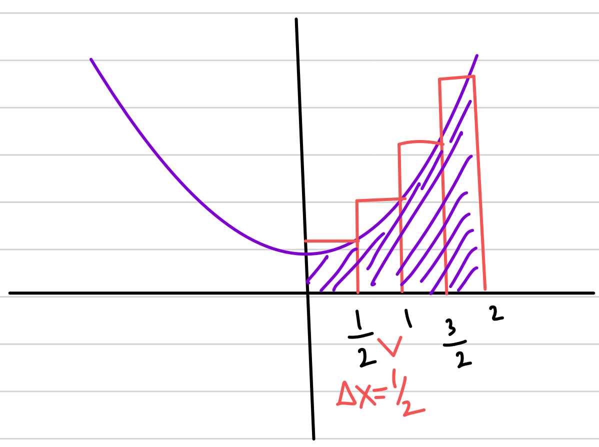


Δx width $= 1$ here

$$\text{Area} \approx \text{Area Rectangle 1} + \text{Area Rectangle 2}$$
$$= \text{Height}_1 \cdot \text{Width} + \text{Height}_2 \cdot \text{Width}$$
$$= f(1) \cdot 1 + f(2) \cdot 1$$
$$= (1^2+1) \cdot 1 + (2^2+1) \cdot 1$$
$$= 2 \cdot 1 + 5 \cdot 1 = 2 + 5 = 7$$

Overestimate

2. $f(x) = x^2 + 1$



Δx width $= \frac{1}{2}$ here

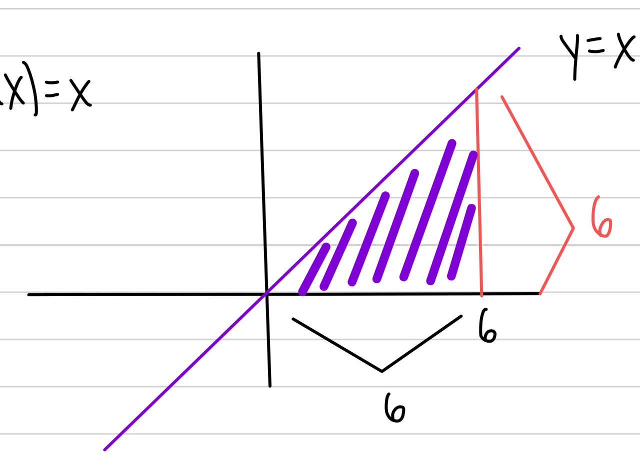
$$\text{Area} \approx \text{Area Rect}_1 + \text{Area Rect}_2 + \text{Area Rect}_3 + \text{Area Rect}_4$$
$$= H_1 \text{ Width} + H_2 \cdot \text{Width} + H_3 \text{ Width} + H_4 \cdot \text{Width}$$
$$= f\left(\tfrac{1}{2}\right) \cdot \tfrac{1}{2} + f(1) \cdot \tfrac{1}{2} + f\left(\tfrac{3}{2}\right) \tfrac{1}{2} + f(2) \cdot \tfrac{1}{2}$$
$$= \left(\left(\tfrac{1}{2}\right)^2 + 1\right) \tfrac{1}{2} + (1^2+1) \cdot \tfrac{1}{2} + \left(\underbrace{\left(\tfrac{3}{2}\right)^2 + 1}_{}\right)^{9/4 + 4/4} \cdot \tfrac{1}{2} + (2^2+1) \cdot \tfrac{1}{2}$$
$$= \tfrac{5}{4} \cdot \tfrac{1}{2} + 2 \tfrac{1}{2} + \tfrac{13}{4} \cdot \tfrac{1}{2} + 5 \cdot \tfrac{1}{2}$$

OR, Can factor out $\frac{1}{2}$ ...

$$= \tfrac{5}{8} + \cancel{1}^{8/8} + \tfrac{13}{8} + \cancel{\tfrac{5}{2}}^{20/8} = \tfrac{5}{8} + \tfrac{8}{8} + \tfrac{13}{8} + \tfrac{20}{8} = \tfrac{46}{8}$$

Overestimate but better estimate

3. $f(x) = x$



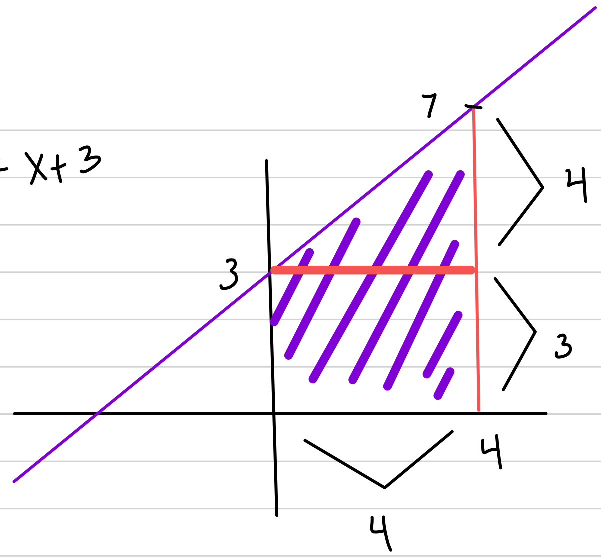
Area Interpretation

$$\text{Area} = \tfrac{1}{2} \text{ base} \cdot \text{height}$$
$$= \tfrac{1}{2} \cdot 6 \cdot 6 = \tfrac{36}{2} = 18$$

Function Value

$f(6) = 6$

4. $f(x) = x + 3$



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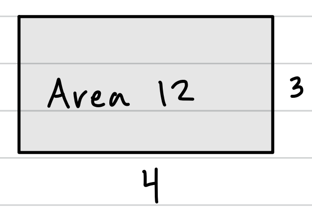

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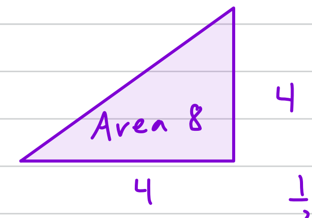
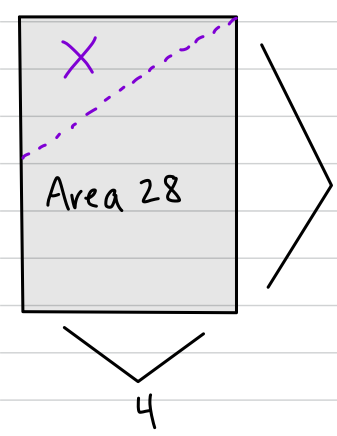


$\frac{1}{2}\, b \cdot h = \frac{1}{2}\, 4 \cdot 4 = \frac{16}{2} = 8$

Total Area $= 12 + 8 = 20$

OR



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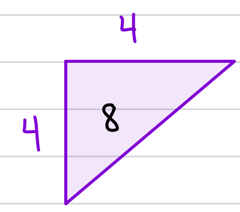

$= 28 - 8 = 20$ ✓

5. $f(x) = x$

$f\left(\frac{5i}{n}\right) = \frac{5i}{n}$ and $f\left(2 + \frac{5i}{n}\right) = 2 + \frac{5i}{n}$

$f(x)$ is the Identity here

6. $f(x) = 3x - 4$

$f\left(\frac{8i}{n}\right) = 3\left(\frac{8i}{n}\right) - 4 = \frac{24i}{n} - 4$

$f\left(3 + \frac{8i}{n}\right) = 3\left(3 + \frac{8i}{n}\right) - 4 = 9 + \frac{24i}{n} - 4 = 5 + \frac{24i}{n}$

7. $f(x) = x^2 + 5$

$f\left(\frac{2i}{n}\right) = \left(\frac{2i}{n}\right)^2 + 5 = \frac{4i^2}{n^2} + 5$

$f\left(4 + \frac{2i}{n}\right) = \left(4 + \frac{2i}{n}\right)^2 + 5 = \left(4 + \frac{2i}{n}\right)\left(4 + \frac{2i}{n}\right) + 5$

$= 16 + \frac{8i}{n} + \frac{8i}{n} + \frac{4i^2}{n^2} + 5$

$= 21 + \frac{16i}{n} + \frac{4i^2}{n^2}$

8. $f(x) = x^2 - 2x + 7$

$f\left(\frac{6i}{n}\right) = \left(\frac{6i}{n}\right)^2 - 2\left(\frac{6i}{n}\right) + 7 = \frac{36i^2}{n^2} - \frac{12i}{n} + 7$

$f\left(-1 + \frac{6i}{n}\right) = \left(-1 + \frac{6i}{n}\right)^2 - 2\left(-1 + \frac{6i}{n}\right) + 7$

$= 1 - \frac{6i}{n} - \frac{6i}{n} + \frac{36i^2}{n^2} + 2 - \frac{12i}{n} + 7$

$= 10 - \frac{24i}{n} + \frac{36i^2}{n^2}$

9. $\lim\limits_{n\to\infty} 3 = 3$ Limit of a constant equals the constant

10. $\lim\limits_{n\to\infty} \cancelto{0}{\frac{1}{\underset{\infty}{n}}} = 0$

11. $\lim\limits_{n\to\infty} 1 + \cancelto{0}{\frac{1}{\underset{\infty}{n}}} = 1$

12. $\lim\limits_{n\to\infty} \frac{n+1}{n} = \lim\limits_{n\to\infty} \cancelto{1}{\frac{n}{n}} + \cancelto{0}{\frac{1}{\underset{\infty}{n}}} = 1$

13. $\lim\limits_{n\to\infty} \frac{n+3}{n} = \lim\limits_{n\to\infty} \cancelto{1}{\frac{n}{n}} + \cancelto{0}{\frac{3}{\underset{\infty}{n}}} = 1$

14. $\lim\limits_{n\to\infty} \frac{2n+1}{n} = \lim\limits_{n\to\infty} \cancelto{2}{\frac{2n}{n}} + \cancelto{0}{\frac{1}{\underset{\infty}{n}}} = 2$

15. $\lim\limits_{n\to\infty} \frac{n(n+1)}{n^2} = \lim\limits_{n\to\infty} \cancelto{1}{\left(\frac{n}{n}\right)} \left(\frac{n+1}{n}\right) = \lim\limits_{n\to\infty} \cancelto{1}{\frac{n}{n}} + \cancelto{0}{\frac{1}{\underset{\infty}{n}}} = 1$

Partner

OR// $\lim\limits_{n\to\infty} \frac{n(n+1)}{n^2} = \lim\limits_{n\to\infty} \frac{n^2+n}{n^2} = \lim\limits_{n\to\infty} \cancelto{1}{\frac{n^2}{n^2}} + \frac{\cancel{n}}{n^{\cancel{2}\,1}} = \lim\limits_{n\to\infty} 1 + \cancelto{0}{\frac{1}{\underset{\infty}{n}}} = 1$

16. $\lim\limits_{n\to\infty} \frac{n(n+1)(2n+1)}{\underbrace{n^3}_{n\cdot n\cdot n}} = \lim\limits_{n\to\infty} \cancelto{1}{\left(\frac{n}{n}\right)} \left(\frac{n+1}{n}\right) \left(\frac{2n+1}{n}\right) = \lim\limits_{n\to\infty} 1 \cdot \left(1+\cancelto{0}{\frac{1}{n}}\right) \left(2+\cancelto{0}{\frac{1}{n}}\right)$

$= 1 \cdot 1 \cdot 2 = 2$

17. $\lim\limits_{n\to\infty} 3 - \left(\frac{4}{n^2}\right) \left(\frac{n(n+1)}{2}\right) - \left(\frac{12}{n^3}\right) \cdot \left(\frac{n(n+1)(2n+1)}{6}\right)$

Repartner

Split-Split

$= \lim\limits_{n\to\infty} 3 - \cancelto{2}{\left(\frac{4}{2}\right)} \cancelto{1}{\left(\frac{n}{n}\right)} \left(\frac{n+1}{n}\right) - \cancelto{2}{\left(\frac{12}{6}\right)} \cancelto{1}{\left(\frac{n}{n}\right)} \left(\frac{n+1}{n}\right) \left(\frac{2n+1}{n}\right)$

$= 3 - 2 \cdot 1 \left(\cancelto{1}{\frac{n}{n}} + \cancelto{0}{\frac{1}{n}}\right) - 2 \cdot 1 \cdot \left(\cancelto{1}{\frac{n}{n}} + \cancelto{0}{\frac{1}{n}}\right) \cdot \left(\cancelto{2}{\frac{2n}{n}} + \cancelto{0}{\frac{1}{n}}\right)$

$= 3 - 2 \cdot 1 \cdot 1 - 2 \cdot 1 \cdot 1 \cdot 2 = 3 - 2 - 4 = -3$

18. $\sum\limits_{n=1}^{n} 6 = \underbrace{6 + 6 + 6 + \cdots + 6}_{n \text{ copies}} = 6n$

OR// $\sum\limits_{n=1}^{n} 6 = 6 \sum\limits_{i=1}^{n} 1 = 6\underbrace{(1+1+1+\cdots+1)}_{n \text{ copies}} = 6 \cdot n = 6n$

19. $\sum_{i=1}^{n}(-3) = -3\underbrace{\sum_{i=1}^{n} 1}_{n} = -3\;n = -3n$ OR// $\sum_{i=1}^{n} -3 = \underbrace{-3-3-3-3-\cdots-3}_{n\text{copies}} = -3\cdot n$

20. $\sum_{i=1}^{n}\left(\frac{6i}{n}-5\right)\cdot\left(\frac{6}{n}\right) = \sum_{i=1}^{n}\frac{36i}{n^2}-\frac{30}{n} = \sum_{i=1}^{n}\frac{36i}{n^2} - \sum_{i=1}^{n}\frac{30}{n}$

$$= \frac{36}{n^2}\sum_{i=1}^{n} i - \frac{30}{n}\underbrace{\sum_{i=1}^{n} 1}_{n}$$

$$= \frac{36}{n^2}\sum_{i=1}^{n} i - \cancel{\frac{30}{n}\cdot n} = \frac{36}{n^2}\sum_{i=1}^{n} i - 30$$

Match

21. $\sum_{i=1}^{n}\left(1+\frac{3i}{n}\right)^2\cdot\left(\frac{3}{n}\right) = \sum_{i=1}^{n}\left(1+\frac{6i}{n}+\frac{9i^2}{n^2}\right)\cdot\frac{3}{n}$

$$= \sum_{i=1}^{n}\frac{3}{n}+\frac{18i}{n^2}+\frac{27i^2}{n^3}$$

$$= \sum_{i=1}^{n}\frac{3}{n}+\sum_{i=1}^{n}\frac{18i}{n^2}+\sum_{i=1}^{n}\frac{27i^2}{n^3}$$

$$= \cancel{\frac{3}{n}\sum_{i=1}^{n} 1} + \frac{18}{n^2}\sum_{i=1}^{n} i + \frac{27}{n^3}\sum_{i=1}^{n} i^2$$

$$= 3+\frac{18}{n^2}\sum_{i=1}^{n} i + \frac{27}{n^3}\sum_{i=1}^{n} i^2$$

match